Three "Open" Methods For Teaching Elementary School Mathematics

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Abstract

It is common for students in their mathematical studies to encounter problems that have unique answers that are uniquely determined. Due to these experiences, students often come to hold the belief that there is only one correct answer to every problem and only one correct way to answer it.

In the Japanese open approach to teaching, mathematical problems are selected that exemplify a diversity of approaches to solving a problem (the process is open) or that have multiple correct answers (the end products are open). There is also emphasis on having students formulate new problems (the ways of formulating problems are open). The teacher can give students experiences in learning something new by combining their previous knowledge, skills, and mathematical ways of thinking.

In this article, I first give examples of the types of problems used in this approach. Then I discuss the assessment, that is, assessing problems, assessing students' work and assessing lesson plans. Finally, the relations between open teaching methods and three curriculums will be discussed.

Three "Open" Methods for Teaching Elementary School Mathematics

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I Examples of Open Problems

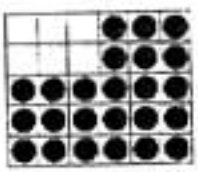
Three aspects of the approach are presented below: open process, open problem formulation and open end product.

1. Open Process

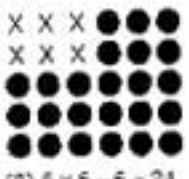
To create interest and stimulate creative mathematical activity in the classroom, the Japanese developed a tradition in teaching to focus on the different ways of solving a problem when the answer is unique —— in this case we say the problem solving process is open. At all grade levels, it is possible to have students put their own natural mathematical ways of thinking to use in approaching a problem. In doing so, students will exhibit a variety of approaches, and these can be shared and discussed in class. Some examples of these problems are given next, along with anticipated or actual responses of students.

Example 1 Chocolates Problem

How many chocolates are there in the box?

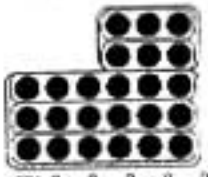


(1) Counting one by one, so 24
(2) Grouping by threes, so $8 \times 3 = 24$

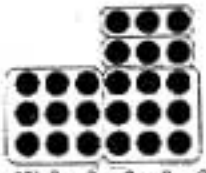


(3) $5 \times 6 - 6 = 24$

(4) $4 \times 6 = 24$



(5) $2 \times 3 + 3 \times 6 = 24$



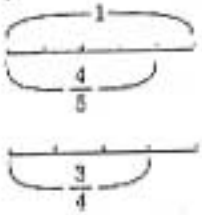
(6) $3 \times 3 + 2 \times 3 = 24$

(7) $3 \times 3 - 3 \times 5 = 24$

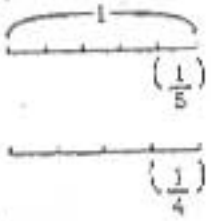
Example 2 Fraction

Which is larger: $\frac{4}{5}$ or $\frac{3}{4}$?

(1)



(2)



(3) $\frac{4}{5} = 4 \div 5 = 0.8$, $\frac{3}{4} = 3 \div 4 = 0.75$ $\Big\}$ $0.8 > 0.75$

(4) $\frac{4}{5} = \frac{8}{10} = \frac{12}{15} \cdots$, $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{15} \cdots$ $\Big\}$ $\frac{4}{5} > \frac{3}{4}$

(5) $\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \frac{16}{20} \cdots$, $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} \cdots$ $\Big\}$ $\frac{4}{5} > \frac{3}{4}$

For each problem above, several approaches to finding the answer are given. The different students' responses can be compared and discussed in terms of their mathematical features and their qualitative differences.

2. Open Problem Formulation

The ways in which problems can be formulated are also open. Students are taught to formulate or pose new mathematical problems from a given problem by using generalization, analogy, the idea of converse, or other ideas, and then solve the newly formulated problems by themselves. When first encountered, problem formulation or posing new problems is novel to almost all students. Below I give two examples of original given problems and several possible or actual problems that students may formulate based on each of them.

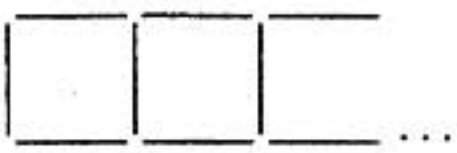
Example 3 Butterfly Problem

Given Problem: There are four butterflies. If three more butterflies arrive, how many are there?

a. Children change the number of butterflies.
b. Children change the objects. In Japanese, the counting system often differs between objects being counted, say, butterflies or cars.
c. When the problem involves "increasing" situation, children change the problem to an "altogether" situation.
d. Children change the addition problem to a subtraction problem.
(Ex.) There are seven butterflies. If four butterflies flew away, how many are there remaining?

Example 4 Toothpick Problem

Given Problem: Toothpicks are placed to make squares as in the figure below. When the number of squares is five, how many toothpicks are used?



To formulate problems,

a. Change the number of squares.
b. Change the number of rows of squares.
c. Change the figure (e.g., from an square to triangles, pentagons, etc.).
d. Write the converse problem.

e. Change from two to three dimensions.
f. A combination of the above changes.

3. Open End Product

Traditionally, problems are so well formulated that answers are either correct or incorrect (including incomplete ones). We regard such problems as "closed" or "complete" problems. In contrast, problems that are formulated to have multiple correct answers are "incomplete," "open end," or "open-ended" (Shimada, 1977; Becker and Shimada 1997). Such problems involve students finding one, several, or many correct answers to one problem, and they may use various methods to find those answers.

We can think about three types of problems with open end product: finding rules or relations, classifying, and measuring. Some examples follow.

(1) Finding Rules or Relations

Example 5 Properties of the Multiplication Table

The number table shown below was produced in accordance with a certain rule. Find as many relationships as possible among the numbers in the table by studying the arrangement of numbers.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

a. The number which results from multiplying a number from the top row and a number from the leftmost column can be found at the intersection of the row and the column.
b. The numbers in a column increase by the same number.
c. Symmetry at the axis of the diagonal (from upper left to lower right).
d. There exist square numbers on the diagonal line (from upper left to lower right).
e. The sum of the numbers in the first column is 55; the sum of the numbers in

the second column is 110, two times the sum of the number in the first, etc.

f. The sum of the numbers before and after a certain number in a row (column) is equal to two times the number.

g. Any four numbers making a square have the relationship that the products will be the same when they are cross-multiplied.

h. The sum of the total number of the second (xth) column and that of the third (yth) column equals to the total number of the fifth (x+y th) column.

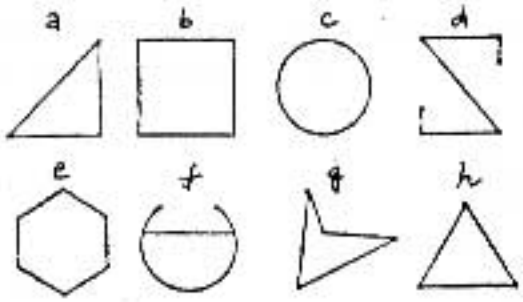
i. Eight times the center number equals the sum of the numbers surrounding the center in a three by three section of numbers.

As mentioned, the purpose of this problem is to have students find as many rules as possible from several points of view. The rules may range from lower to higher levels.

(2) Classifying

Example 6 Classifying Problem (Plane)

There are eight plane figures as follows.



Classify using common properties among eight plane figures.

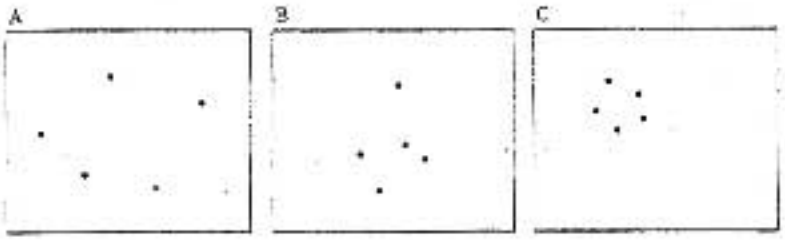
- straight edges a b d e g h
- circular edges c f
- simple closed figures a b c e g h
- triangles a h
- quadrilaterals b g
- lines of symmetry a b c e f h
- point of symmetry b c d e
- regular polygon b e h
- concave polygon g
- diagonals b e

etc.

(3) Measuring

Example 7 Marble Problem

Three students, A, B, C,throw five marbles that come to rest as in the figure below. In this game, the student with the smallest scattering of marbles is the winner. To determine the winner, we will need to have some numerical ways of measuring the scattering of the marbles.



(a) Think about this situation from various points of view and write down different ways of indicating the degree of scattering.
(b) Which way appeals to you?

- Measure the area of the polygonal figures formed by joining the points.
- Measure the perimeter of the polygonal figures formed by joining the points.
- Measure the length of the longest segment connecting two points.
- Add the lengths of all segments connecting two points.
- Add the lengths of the segments connecting one fixed point with all other points.
- Measure the radius of the smallest circle that includes all points.
- Calculate the standard or average deviation using a coordinate system.

II Assessment

An open approach encourages many students' productions in the class for ordinary students as well as bright or highly motivated students. Assessment ideas are also numerous and flexible. Assessing problems, assessing students' works and assessing lesson plans are discussed.

1. Assessing Problems

In general, problems, like those above, are assessed from the following perspective before developing a lesson plan:

(a) Is the problem rich in mathematical content?
(b) Is the level of mathematics appropriate for the students?

(c) Are there features that lead to further mathematical development and provide opportunities to offer original and insightful observations?

2. Assessing Students' Works

(1) Assessing Problem Formulations

As formulating problems may at first be difficult for students to learn, assessing students' problem formulations can be challenging for teachers to learn. Before teaching a lesson, teachers often prepare a list of problems that students might formulate. This list is part of the analysis of the problem and helps to organize a discussion of the formulated problems. The student work can be evaluated by considering the following three factors: the number of problems formulated, how the students formulated their problems, and the types of mathematical concepts embedded in the new problems.

For example, students may formulate new problems by changing a number, changing an object, changing an object and a number, making an analogy, using the converse of the given problem, combining several problems. Different types of problem formulation demonstrate different types of thinking.

(2) Assessing Solutions

Teachers can collect assessment data by examining students' worksheets that are collected at the end of a lesson, by purposeful scanning of students' work while they work on the problem, and by observation during students' discussion of their production. These observations can be recorded. Thus, during the lesson, the main means of assessment is by teacher observation. After the lesson, the teacher can examine students' worksheet. The approach is a formative one during and after the lesson, but when the results of the assessment are accumulated over time, they become information for a summative assessment of students.

An assessment scheme that has been used by us has the following four features.

a. fluency b. flexibility c. originality d. elegance

3. Assessing Lesson Plans

Before discussing the assessment of lesson plans, I would like to say a few words about lesson plan development.

Lesson plans according to the following scheme were organized:

a. Introducing the problem or topic
   The teacher presents or poses a problem (pertaining to an objective) on the overhead projector, chalkboard, or using a poster.
b. Understanding the Problem
   The teacher ensures through discussion and soliciting questions that

students know what is expected of them before they begin work.

c. Problem Solving by the Students

Students are given a worksheet with the problem written on it and space to write their work. They work individually or in small groups. Students' natural ways of thinking are encouraged, thereby "drawing out" a variety of responses. The teacher moves among the students purposefully scanning their work and selecting thinking approaches or answers concerning to the objective that will be discussed with the whole class.

d. Comparing and Discussing

Individual students (or groups) write their approaches or answers on the blackboard (or overhead projector) for all students to see. The teacher then guides a discussion of the responses, comparing their mathematical features and quality.

e. Summary of the Lesson

The teacher plays an important, cruciai role in pulling together the outcomes of discussion and lesson as they relate to the objective.

In developing a lesson plan, then, the following points need to be addressed:

- Students' expected responses are listed and grouped according to their mathematical features.
- The purpose of the problem is made clear.
- Students are given the help needed to understand the problem as well as what is expected of them.
- Students are given time to fully explore and discuss the problem.
- Students record their responses on a worksheet or in their notebooks.
- Certain students write their work on the chalkboard for all to see.
- Students' work is compared and discussed.
- The teacher summarizes what has been learned in the lesson.

With the open approach, a teacher may not anticipate all that will happen in the classroom.

Besides the lesson-plan scheme discussed above, other related schemes have been developed. For example, the following lesson plan outline shows a developmental treatment of problems (also called "problem to problem") where, instead of a single problem, a sequence of problems is employed to meet an objective.

- Solve the given problem.
- Compare and discuss the methods of, and the solution(s) to, the problem.
- Formulate new problems by changing parts (conditions) of the given problem.
- Propose new problems to the whole class.
- Discuss the new problems and classify them.
- Solve problems selected by the students and teacher.

- Solve problems formulated by the students.

When we think about assessment, above viewpoints must be considered.

A sample problem is given below (Hashimoto and Becker, 1999, p.115).

Parks Problem

There are three parks, A, B, and C, in which many boys and girls are playing. The areas of the parks A, B, C are , respectively, 500 ㎡, 500 ㎡, and 300 ㎡. The number of children playing in these parks is, respectively, 40, 30, and 30. Which park is the most crowded?

III Relation to Curriculum

The so-called three curriculums, that is, the intended, implemented and attained curriculums as defined by the International Mathematics and Science Study, are related to "open methods" discussed in this paper.

Open problem formulation can be seen in elementary and junior high school textbooks and in National Assessments, given in 1994 and in 2002 for elementary school mathematics and in 1995 for junior high school mathematics.

A book (konioujita shidou nikansuru shidoushiryou in Japanese) on teaching elementary school mathematics was published in November 2002. Ten thousands copies were sold immediately. Before it was published, the Ministry of Education, Culture, Sports, Science and Technology distributed the book to all elementary schools in Japan and all the boards of education. The number of schools is approximately 25000. Some examples about three open methods are included in this book. The balance among three curriculums is very important.

IV Conclusion

To foster students' mathematical thinking, it is crucially important to allow them to think freely ——that is, to use their own mathematical ways of thinking. This is at the heart of the open approach. Although Japanese mathematics lessons involve whole-class instruction, it can be seen that mathematics teachers indeed address individual students' activities or needs. In particular, opportunities are provided for students with great mathematical potential to exercise their creative abilities and share their insights with other students.

Even if a problem has only one solution, there may be several ways to find the solution. Indeed, this is often the starting point to developing students' different ways of thinking. In addition to using several ways to solve a problem with a unique answer, the open approach employs problems that have different correct solutions and allows students to formulate or pose problems of their own. Lesson

plans are geared toward exploring these open problems, and the observation of student during lessons and analyses of students' worksheets after the lessons are important vehicles for carrying out the assessment of students' learning.

I think the most important point is how classroom teachers implement their curriculum. In other words, much depends on the teacher's philosophy of mathematics teaching. Some teachers emphasize the value of the discussion between children and of children's ideas. Such kind of classroom teacher walks around the desks and cares about what and how children are thinking.

References

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Title : **Three "Open" Methods for Teaching Elementary School Mathematics**

Presenter : Professor Yoshihiko Hashimoto

Date & Time : 13 August 2003, 9.00 a.m. – 10.00 a.m.

1. Content of the Paper

1.1 When solving a problem, there are diversity of approaches (the process is open) and the end products are different with multiple correct answers.

1.2 Students are encouraged to pose new problems and are allowed to apply their own natural mathematical ways of thinking when approaching a problem. Also, students exhibit a variety of approaches that could be shared and discussed in class.

1.3 Teachers should provide situations for students to relate previous knowledge, skills, and mathematical ways of thinking to new knowledge.

1.4 Students' works are evaluated using three factors: number of problems formulated, how students formulated their problems, and the type of mathematical concepts embedded in the new problems.

1.5 Teachers should examine students' worksheets, going through students' work, and observe students' during discussion time.

2. Discussion

2.1 *Assoc. Professor Dr. Jamaludin Md. Ali of Universiti Sains Malaysia*, enquired whether Malaysian students have background knowledge in arithmetic (as Malaysian students), whether Japanese students stress more on tools or on concepts and whether they write down and think aloud when solving problems?

Answer
Japanese students acquired basic concepts in using positive and negative numbers. Emphasis is given both to tools and concepts. At the beginning, students usually do not know there are many correct answers to a problem. Students are then given the chance to freely express themselves in tackling the problem given by writing them down and by thinking aloud through presentations. The open-ended approach is already introduced to 25,000 elementary schools.

2.2 *Mr. Norjoharuddeen Mohd. Nor of SEAMEO RECSAM Regional Centre for Education in Science and Mathematics, Penang, Malaysia* commented that

the open-ended approach is not being used in Malaysia and enquired how we can get teachers to use this approach.

Answer
In Japan, open-ended questions are only used in high school level entrance exam and in some prefectures depending on interested individuals. Since 2002, teachers are given guidebooks on Open-ended Approach so that they are able to use them in classrooms.

2.3 *Ms. Jamaliah Hashim of Raja Melewar Teacher Training College, Negeri Sembilan, Malaysia* commented that in Malaysia, the approach is not fully supported. In view of this, teachers should make the effort to include this approach in their mathematical activities.