

## **NESTING FEATURES OF DEVELOPING TEACHERS' PERSPECTIVES: A LESSON STUDY PROJECT FOR PROSPECTIVE TEACHERS IN MATHEMATICS WITH HISTORY AND TECHNOLOGY**

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*For teacher education with technology, we should consider many questions but there are no answers without focusing on the parameters. Here we discuss about a case for developing a good teacher's perspective through Lesson Study in terms of Japanese meaning with technology. Firstly, we define desirable teachers' perspectives. Secondly, we focus on the function of technology and history for teacher education. Thirdly, we analyze the case for explaining the developing process of teachers' perspectives in it.*

### **I. Development of Teacher's Perspectives 'Kodomo wo miru me' for Mathematics**

In the APEC meeting 'Innovative Teaching Mathematics through Lesson Study' in January 2006 at Tokyo, Catherine Lewis (2006) talked about her experience on Lesson Study in her keynote lecture as follows: (In her Lesson Study project) A U.S. teacher said as follows: "Before the Lesson Study, we had talked about multiple intelligence, constructivism and so on, but never talked about each subject matters of teaching. In the Lesson Study project, we began to talk about subject matters, why we teach them, how we teach them and what students learn from the lesson". In Tokyo's session, majority of participants may feel that this episode is not just for U.S. but for all countries. In the in-service teacher training programs, mathematics educators used to teach the theory of mathematics education. A comment from a teacher implicates that we teach theory and policy of curriculum and failed to teach them with subject matters. Multiple intelligence theory made us notice desirable competency which is not developed by one subject. In curriculum, teachers are expected to develop it through their lesson through teaching contents.

Constructivism theory promoted our awareness of the importance of listening students' ideas because students construct their knowledge by themselves. In teaching context, teacher's listening is not passive action such as only hearing but positive action (Arcavi & Isoda, to appear). Good lessons based on constructivism expect student-centralized lesson and the roles of teachers to conduct students' activity for their learning. In this context, listening activities by teachers are aimed to think about and find the way how to develop students' ideas to sophisticate or elaborate with others in their classrooms.

Lesson Study is an authentic activity for enabling teachers to conduct their classrooms. It includes discussions of subject matters, why they teach, how they teach and what students can learn.

Catherine also noticed her experience as follows; One teacher said, "I developed the eyes (teacher's perspective) to look at students and subject matters "Kodomo wo miru me". Now, I am well aware of my responsibility for my lesson. In the lesson study with other teachers, I preferred the more challenging lesson such as with Open-ended problems. When I found that students can challenge such difficult problems, I recognized self-confidence in my lessons". Catherine mentioned that teachers developed the ability to listen to students' ideas such as 'Kodomo wo miru me' but it is not only hearing (See such as Catherine

Lewis 2002). Because they developed good teachers' perspectives, they can say the development of eyes for understanding students and want to challenge the lesson with Open-ended problems and feel self-confidence through conducting the lesson.

Based on Japanese ideas of Lesson Study, teacher's perspective 'Kodomo wo miru me' is explained as the following (Isoda, Stephens, Ohara and Miyakawa, to appear): In Lesson Study, teachers discuss about the subject matter before the lesson. Teachers share responses (including misunderstandings) from students in the past lessons, have a lot of expectations about students' ideas and prepare their questions to extract students' ideas and their reaction against students' ideas. At the same time, teachers also expect that students' ideas will be more than their expectations. If students' ideas are within expectations, it is easily understandable for the teacher. Even if not, it is also within their expectations because it is a good chance for them knowing unknown ideas from students.

In teacher education, it is necessary to develop teachers for stepping up from listening to conducting. What necessary conditions for stepping up are and what kinds of processes are important for it even if there are no sufficient conditions. For example, some good teachers teach students the value what is important for life in any time and believe mathematics teaching is a part of the value education (Alan Bishop et al. 2003). Some novice teachers act differently between mathematics class and homeroom class. They worry how to solve and how to teach mathematics problems in every lesson but in homeroom activity, they try to push students' decision making. Through the experience, we can expect novice teachers to develop themselves to integrate their teaching contents and value. In this paper, the conditions and the processes are discussed as a case study.

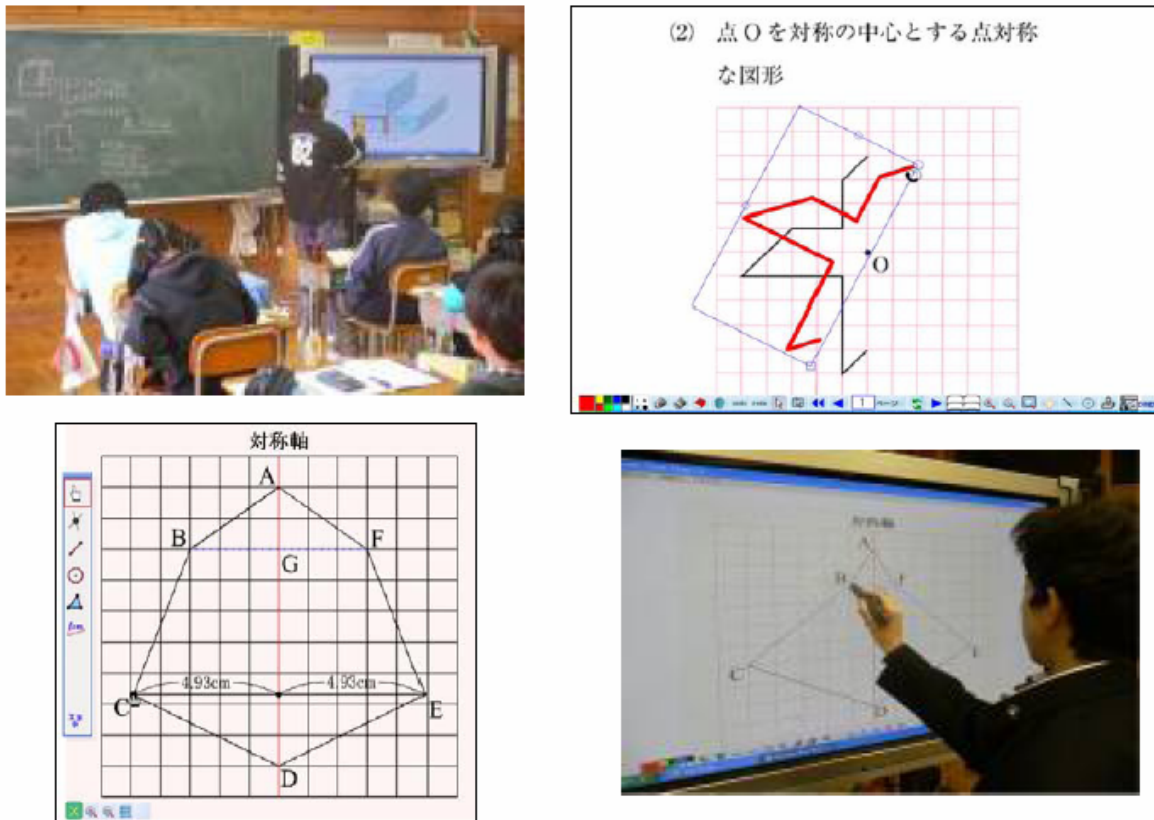
## **II. Technology and History for Knowing Mathematics Differently**

### **1. Minimum necessity to use technology for teacher education.**

e-Learning is a current technology movement in education. Developing knowledge bank with learning management system is a trend. Equipment in schools and environment of internet are well known obstacles in general. But even if equipped, each teacher's belief of mathematics is an obstacle because mathematics is already embedded in physical or psychological tools such as papers, pencils and calculations. It is not easy to change teachers' beliefs because if we change tools then we have to change our mathematics itself. If we think their beliefs as an obstacle, we can not change. On contrary, If we recognize that each technological tool has it's own way of knowing mathematics differently, technology supports teacher educators to teach school mathematics differently and it may be a cue for next step.

For example, in mid of 90's, I engaged in in-service teacher training summer course to use a Graphing Calculator, Computer Algebra System and Dynamic Geometry Software during 5 years. The number of participants is more than fifty every year and half of them are repeaters. They enjoyed mathematics with technology, got the skill how to use and develop lesson plans for their classroom. But most of them did not use computers and graphing calculators in their classrooms because mathematics had been taught without technology and most subject matters in textbooks are not necessary to use technology. Between lines in textbooks, there are many things that should be taught. In a simple algebraic calculation from a line to a line, there are things which should be explained. Teacher can not alternate it to technology. In 90's, most of the technology developed as the environment and some mathematics educators believed to alternate the hidden aims in textbooks with

technological environment. Indeed, we are now in a process of alternating textbooks to e-textbooks. The difference is that e-textbooks are a kind of textbooks. Teachers do not need to learn the commands how to use and integrate their aim of teaching with technological environment in the classroom (See Picture 1, Isoda et al. 2005).

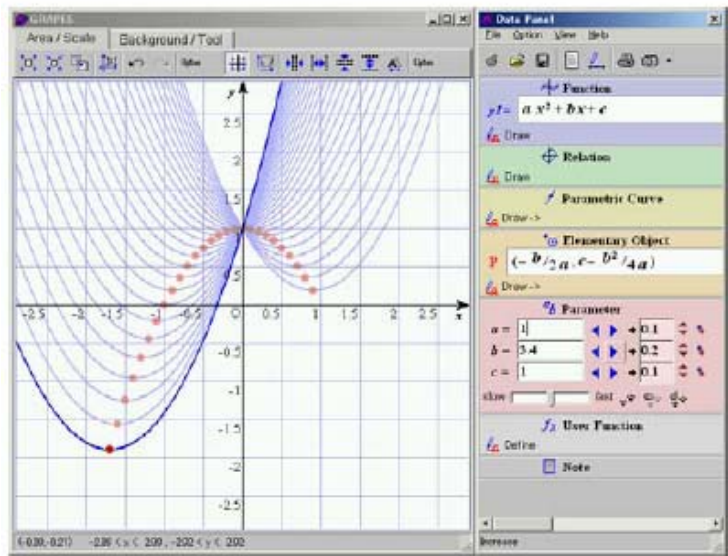


**Picture 1.** Using e-textbook with Interactive Board in classroom (Isoda et al. 2005)

What is obscure for me is that why many teachers had participated in summer courses even if they did not have a wish to alternate. I could say that they enjoyed knowing mathematics from different ways with technology. They enjoyed explorations of mathematics via technology. For example, if we draw graphs of  $y = ax^2 + bx + c$  by fixing two parameters from a, b and c and changing one remained parameter regularly (Picture 2), we can find the role of each parameter, a, b or c which is never known by algebraic deduction to  $y = (x - \alpha)^2 + \beta$ .

Even if teachers did not have a chance to use computers or graphing calculators in their schools, exploring mathematics with technology in summer course is an enjoyable experience for them because it is the chance to know their known mathematics differently. If we use unknown technology, teachers can explore their school mathematics as unknown.

If we say that minimum necessity is needed to use technology in teacher education, we can say that it gives prospective or in-service teachers to explore school mathematics as a really new one. Teachers can re-experience their mathematics like students who learn from the beginning. Even if it is impossible because they already know, knowing differently is meaningful. If teachers know how to enjoy mathematics, it supports teachers enabling students to enjoy mathematics.



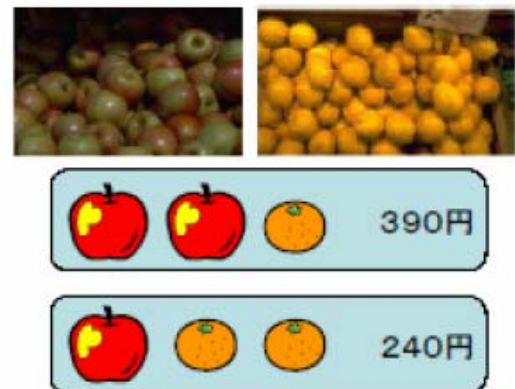
Picture 2. Grapes (Isoda et al. 2005)

## 2. Any Technology is innovative for knowing mathematics differently.

When we think about a function of technology in mathematics teacher education knowing mathematics differently, it is not necessary to focus on innovative technology because if we change technological or psychological tools (James Wertsch. 1991) we know mathematics differently. For example, if I have a card written with the number 2 in my left hand and I have a card with the number 6 in my right hand, and ask pupils to read cards, they must read two and six. If we bring closer both cards and ask the same question, what will happen? Pupils may begin to read twenty six. Even if we know that is the definition, we re-aware the difficulty and marvelous features of base ten system. Number Cards enable us to re-aware mathematics.

In elementary school mathematics, we usually use concrete materials for understanding. It is supported by not only Piaget' constructivism but also the theory of embodiment (George Lakoff, Raffael Nunez., 2000). For prospective teachers training, concrete materials are usually reused for teaching the methods of teaching because prospective teachers forgot how they learned content but prospective teachers enjoy like students before knowing it as the methods. For example,

in picture 3 (MEXT, 2002), please find the price of an apple and the price of an orange posed with the picture without simultaneous equations. If you can solve it by operation of apples and oranges, you can enjoy unexpected explanation of the algebraic solution of simultaneous equations. Prospective teachers can recognize algebra as generalized operations of concrete objects.



Picture 3. How much each?

What implicates from these three examples here is that mathematical awareness is given with tools. Any technology for mathematics can be innovative for knowing mathematics differently.

### 3. Mathematics history as tools for cultural awareness

For knowing mathematics differently, mathematics itself can be useful. Indeed, mathematics is a psychological tool as for mediational means from the view point of Vygotskian theory (James Wertsch, 1991). History of mathematics itself is another mathematics when comparing with the current school mathematics. For mathematics teachers, I have been developing a web site in mathematics and history (See Picture 4. Isoda). It is not the web site of history itself. It's aim is to know mathematics differently and the origins from history. Most of contents are inspired



**Picture 4.** Mathematics History Museum by the Lesson Study Project (Isoda2005)

from historical texts in mathematics but with added educational view points. For example, in picture 4, it explains how to use sextant which was used for navigation before the age of radar and GPS. It tells us how high school mathematics was useful and necessary. A case study described in the next chapter is the Lesson Study Project that developed this website.

### **III. A Case Study of Developing Teachers' Perspective 'Kodomo wo miru me'**

#### **1. The introduction of Lesson Study in Japanese teacher education**

It is difficult for prospective teachers to think like experienced teachers even if they take classes on a particular academic subject or on materials study. Thus, in teacher education programs in Japan, prospective teachers engage in micro-teaching exercises in which they engage in role playing, alternately playing the role of the teacher and the student to acquire the perspectives of both teacher and learner. They also participate in teaching internships of one month during which they do on-site training in an actual school. This allows students to become familiar with the cyclical Lesson Study process of researching materials, conducting Study Lessons, and holding feedback meetings to facilitate improvement. In the final week of their teaching internships, prospective teachers invite their advisors from the university to participate in their own Lesson Study project at the school.

#### **2. A case study of Master Program in Education, University of Tsukuba**

Becoming teachers by obtaining their Rank 1 Teaching Certificate in a master's degree program are trends in Japan. Each university's master's degree program offers its own excellent and distinctive teacher's education programs. Teacher education programs that cultivate the ability to lead practical and useful educational research are especially welcomed by teachers, the board of education, and the Ministry of Education, Culture, Sports, Science and Technology.

The Mathematics Course of the University of Tsukuba Master's Program in Education, which aims to train teachers for high school and beyond, addresses both pure mathematics and mathematics education. In the two year master program in education, we intend to develop leading teachers in mathematics education in school or university based on the tradition of *ecole normale* from 1873. Based on the image of leading teachers, following conditions are expected in this case study: 1) Good teachers can lead Lesson Study in their school, 2) Good teachers can teach other teachers how to use technology in mathematics from the beginning of his work, and 3) Good teachers can lead in the society of mathematics education.

In their first year of two year program, graduate students (prospective teachers) develop original mathematics teaching materials, conduct a three-hour Lesson Study project and write the research report for describing students' achievements. The project is done as a part of mathematics education class with six credits.

##### **2-1. Aims and schedules on the Lesson Study project:**

The Lesson Study project aimed to develop materials for giving high school students cultural awareness in mathematics, improve their attitudes and brief in mathematics by conducting lessons, and to demonstrate the educational value of the developed materials. The schedule to engage in the Lesson Study in the school year 2001 was as the following;

Phase 1) Transition period (almost April – June): Teacher educator (project director) explained first-year students a year plan of the project and explained what kinds of activities were expected. Second-year students in master program who engaged in last year's projects conduct new first-year students' classes to review the activities from their

actual lessons on the previous year's project. First-year students learned how to use the computers in their Lesson Study from second year students and began the project.

Phase 2) Reading of historical sources in mathematics (almost July – August): Students read historical textbooks (English readings or Japanese translations of primary sources) for excavating teaching materials and *A History in Mathematics Education* (John Fauvel, Jan Van Maanen. 2000) for learning the educational value and teaching methods of mathematics history. Teacher educator supported their reading, made clear interesting points when compared with today's mathematics and excluded the misinterpretation originated from reading mathematics history books with today's mathematics such as Bourbaki.

Phase 3) Subject matter development (almost September – November): Students developed subjects from historical texts, conceptualized lessons, established aims and goals, and developed teaching materials such as textbooks using original (or English translation) texts, slides and activities with computer. Teacher educator helped to find interesting materials from historical texts and supported students to develop structures of textbooks and lessons.

Phase 4) Lesson implementation (almost November – December): Students conducted the lesson. Teacher educator supported students to expect classroom students' activities, especially classroom students' responses and how teachers can use the response. Teacher educator also supported how to use classroom equipments such as projecting students' notebook activities to the screen for sharing students' ideas in the classroom.

Phase 5) Report preparation (almost December – February): Students wrote their research reports, created their web site. Teacher educator supported their references depending on their research problems and also supported their preparations for presentations among the mathematics education society.

#### **IV. Analysis of the Case**

##### **1. Analysis of the prospective teachers' experience through the project**

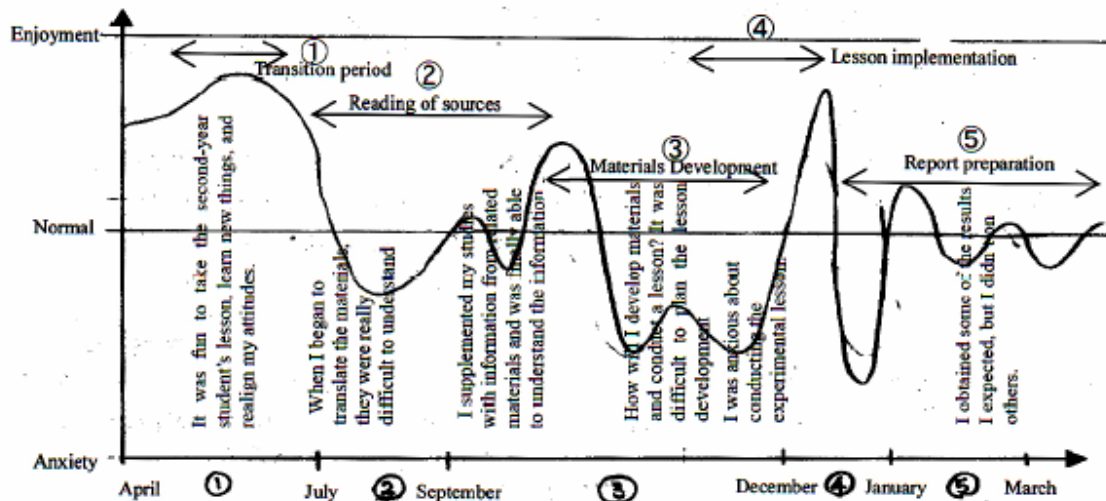
Fourteen prospective teachers in master program participated in the project at school year 2001. After the phase 5, the researcher asked to represent how they changed through the project into the graph of emotions (see Appendix): The x axis of the graph is the time and the y axis is decided by each person, prospective teacher, for representing his/her own emotional change. Each person divided the graph by the periods for describing his/her emotional changes and the graph was explained with the periods by him/her. Thus, up and down of each graph is interpreted by each person's commentaries.

Even if each person's y axis meaning is very different, the phases are well reflected on their graphs (see Appendix: The periods ①~⑤ are rewritten in relation to Phase 1~5, not as same as original periods written by the persons.). In relation to the phases, graphs were categorized as follows: Like the graphs of Appendix 1, two persons' emotional changes are clearly related with the phases. Like the graph of Appendix 2, two persons' emotional changes did not exist phase 1 but other phases are matched with the graphs. They did not recognize phase 1 as a part of project because it was lectured by the second year students. Then, those four persons are clearly related with the phases. Like the graph of Appendix 3, three persons drew their growth of emotion and the highest emotional response is at the lesson implementation phase 4. Like the graph of Appendix 4, three persons connected Phase 2 and Phase 3 because they felt a very strong interest to read historical text as different mathematics and found their original subject matter

for their Lesson Study from their readings. Like the graph of Appendix 5, two persons drew a valley at Phase 3 because they could not easily develop appropriate subject matter for teaching in classrooms. Other two persons' graphs are not clearly related with phases: One of them drew a gradual going up the graph and specially grew up at Phase 4 because he/she finally found strong mathematical interest in his lesson content. Another person drew just down after phase 1 because he/she chose the most difficult text, and felt strong difficulty in reading. He/She did not understand it well at the lesson implementation. He/She commented these kinds of mathematics are very far from school mathematics. All fourteen persons described their first impressions of projects in Phase 1 as interesting activity because they did not know school mathematics with historical text and how to use technology in mathematics. At the same time, even if teacher educator and second graders explained difficulty to read historical text and to develop subject matter from it, they could not imagine what they are and how hard they are to do.

## 2. An interpretation of a case

Even if we can analyze most of graphs in relation to phases, each prospective teacher's experience is very different. The explanations of periods described by each person are just their experience. Following figure is translated in English from one of Appendix 1. Handwritten numbers of ①-⑤ on the x axis are original descriptions of periods and they match to phases, clearly in this case. Here we interpret this person's emotional experience in the following way (Masami Isoda, 1998, 2000, Maitree Inprasitha, 2001): Depending on emotional theory by George Mandler (1984) based on the Piagetian cognitive model, emotional arousal is related with obstacles and challenges, and results such as overcoming obstacles give positive emotional feed backs. This cognitive cycle until reflection is also reasonable from the educational meaning of experience described by John Dewey. Based on Mandler's meaning of emotional change, we can interpret one down-up in the graph recognized as a strong experience.



**Picture 1.** A Case of one prospective teacher's experiences in the project

In this case, we analyze personal experience as follows: In period ①, this person (P) felt fun but did not have strong experience. P participated as a student in second year students' lessons and just enjoyed to learn last year's project. In period ②, there are two strong experiences (two down-ups). P began to read historical text and met the difficulty. P got some understanding of the text but did not understand it well. Then P found related two Japanese translation books and other supplementary books for trying to understand deeply. In period ③ and ④, there are intersections because P continued to develop materials during lesson implementation. P did not know how to develop materials from historical text



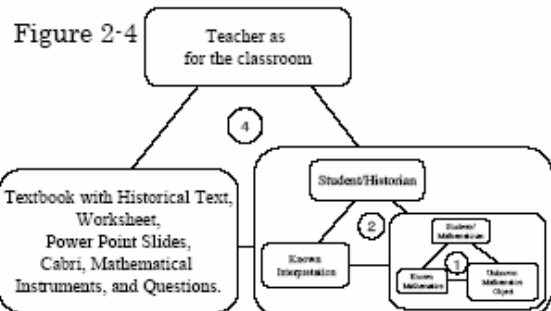
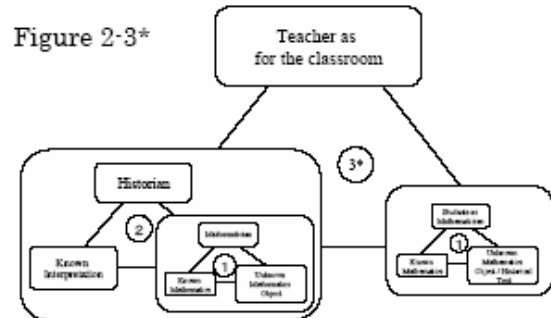
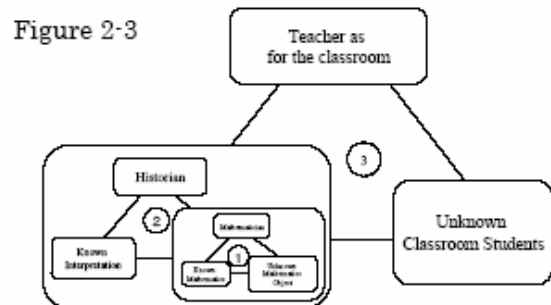
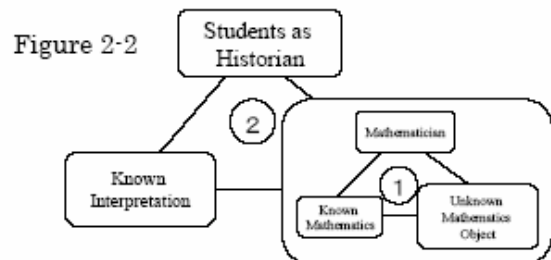
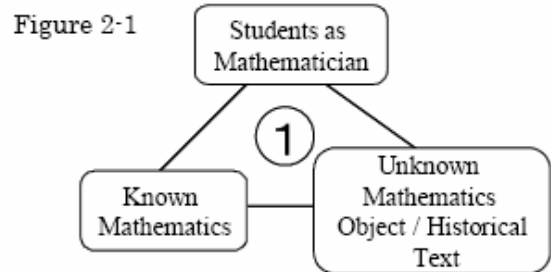
but finally P developed: the strong experience of period ③. P felt anxiety to conduct the lesson but P implemented: the strong experience of period ④. In period ⑤, there is a deep valley after lesson implementation. It is a strong experience because P did not know how to write the report of the lesson. Next small down-up is developing the web site and P did not know the way also.

### 3. Didactical meaning of each phase for prospective teacher education

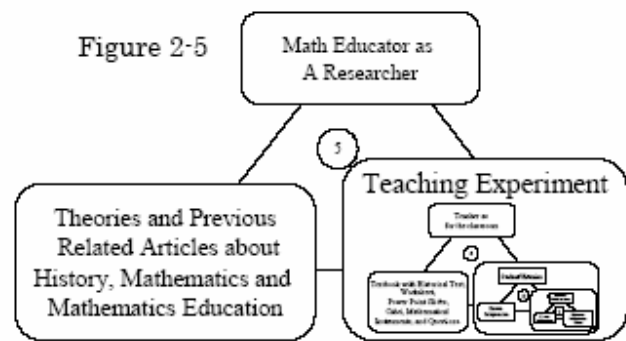
Even if there are two cases which did not well change the graphs in relation to phases, other twelve cases' graphs were explained in relation to phases. Their comments such as the ones seen in the case of figure 1 implicated each phase's didactical meaning for prospective teacher education. For clarifying didactical meaning of phased based on their comments, we would like to framework for interpretation of these data. Hans Nilse Jahnke (1994) used double circles for explaining historian's activity 'Hermeneutics' in mathematics. First circle represents mathematician's activity on history and second circle represents historian's activity such as interpreting historical texts and asking why mathematicians did so. His model well represents the difference of mathematician's perspective and historian's perspective. Jahnke's double circles explain an activity of Phase 2. Here, we would like to expand his model to the field of teacher education for explaining nesting features of developing teachers' perspective 'kodomo womiru me' in the case of this Lesson Study project.

Figure 2-1 explains Phase I activity. Prospective teachers who are participating in the project enjoyed past project's lesson as students. They explored unknown mathematics originated from historical textbooks but reconstructed with educational questions by known mathematics.

Figure 2-2 explains phase 2 activity. They began to interprehistorical texts with known interpretations and were astonished with their differences when compared with today's mathematics. Figure 2-3 explains phase 3 activity.



They began to develop subject Matter. Before the project, they had experience of teaching with existed textbooks and it is the first experience for them to develop the textbook of totally new subject. From historian's activity on figure 2-2, they have to develop students activities with questions for the interpretations of textbook and they have to develop their aims of their lesson study project through thinking about what students can learn from their developed activities (figure 2-3\*). It is very difficult for them because of their past experience of mathematics teaching is only related with mathematical problems but in this project, they have to make historical questions at the same time. Figure 2-4 explains Phase 4 activity. Finally, they had developed materials at phase 3 and then, they tried to conduct students' activities like mathematicians and historians. Figure 2-5 explains Phase 5 activity. They reflect on both of the teaching experiment of Phase 4 and all process of the project and redefine their research questions depending on what they did and analyze it with references.



Based on the analysis, we conclude the following didactical meanings on each Phase for prospective teacher education.

Didactical Meaning of Phase 1: It functioned to know the activity in the lessons through enjoying lessons in past projects like students. Even if teacher educator and second graders explained what the project is and what is necessary to do, such as questionings to classroom students, students, prospective teachers, could not imagine really the meaning because they still work as students who participate in the lessons.

Didactical Meaning of Phase 2: It functioned to know historian's activity such as constructing the meaning through the interpretation of historical texts. Many students felt difficulty to read historical texts at first, then they were astonished with the difference between today's mathematics and historical mathematics.

Didactical Meaning of Phase 3: It functioned to know developing subject matter as for students' activity with historical text and technology. Some students met strong difficulties for developing classroom materials. At the beginning, many students could imagine the textbook of mathematics history and could not develop educational questions through which students can explore historical texts.

Didactical Meaning of Phase 4: It functioned to know conducting the lessons. Many students were scared to conduct. For knowing how to, they practiced with each other before their lessons and expected students' activity based on their questions and reactions from students.

Didactical Meaning of Phase 5: It functioned to know how to write the research paper based on their teaching experiments.

#### 4. Conclusion: A nesting feature of developing teachers' perspectives

These didactical meanings with figure 2-1 to 2-5 illustrate the process how prospective teachers possibly develop teachers' perspectives in this Lesson Study project. In this project sequence, phases are constructed like nesting structures. Every teacher's education subject matter functioned to use previous experiences from different perspectives. For enhancing different meanings of perspectives, we use the word 'role' as follows.

- Role of Phase 1: Like mathematician
- Role of Phase 2: Like historian
- Role of Phase 3: Like textbook author
- Role of Phase 4: Like master teacher
- Role of Phase 5: Like math-educator

We conclude that the case treated various teachers' perspectives such as mathematician, historian, textbook author, master teacher and math-educator. The sequence of Lesson Study project has nesting structures to reflect previous activity from other view points in roles. This process illustrates one of possible way to develop teachers' perspectives. Arcavi, A., Isoda, M. (to appear). Learning to listen: From historical sources to classroom practice.

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Appendix

