

HOW TO DEVELOP MATHEMATICAL THINKING

Shizumi Shimizu, University of Tsukuba

1. Thinking mathematically and mathematical thinking

(1) Idea of Mr. Kenzo Nakajima

Who introduced mathematical thinking into the Course of Study revised in 1958 as aims of mathematics in Japan. Creative activities to be good for mathematics nearly equal to 'thinking mathematically' Mathematical thinking in the Course of Study revised in 1958; aims of elementary school mathematics.

In the aims, mathematical thinking located in the two phases

Mathematical thinking as results created by students

Mathematical thinking as tools students use adequately

There was developing of a scientific attitude in the background.

Mathematical thinking as one point of view of evaluation (after 1970's)

→ development of mathematical thinking

→ meaning of 'development' became ambiguous

→ need to realize the two phases of mathematical thinking again

(2) Idea of Mr. Shigeo Katagiri

His life work is to analyze and classify mathematical thinking from 1960's

Mathematical thinking consists of mathematical idea, method, and attitude which support thinking mathematically

2. Developing student's mathematical thinking in classroom

(1) Putting student's activities in the center of classroom and these activities to be creative or inventive for students

A lesson (classroom) develops mathematical thinking by students' problem solving. Teachers guide and support their activities.

(2) Creative activities (problem solving) should be meaningful both for students and teachers.

We try to analyze the elements and structures of mathematical thinking and to help students acquire them.

3. Thinking mathematically

(1) Students' independent activities

Engaging oneself, not other people's activities

(2) Motivations and phases of activities

Engaging mathematical activities according to phases of them adequately

Motivations and phases

· from Need in life, Explanation of phenomena

→ Using mathematics

Considering or judging by using mathematics

- from Intellectual curiosity, Pursuit of mathematical beauty

→ Creating or discovering mathematics

Thinking creatively or extensively and discovering or inventing new facts, skills, ideas etc. By relation with mathematics of experience by everyday life and having learned already.

- Supporting using and creating mathematics, and from pursuit conclusive evidence or enrichment

→ Explanation or verification

Necessity for understanding by oneself, persuading other peoples sharing results each other, and refining them better

4. Observing classroom activities

5th grade sum of interior angles of polygons

5. Some points of view for improving math classes

- (1) To help students make thinking mathematically a custom
- (2) To represent students' inner process of thinking mathematically
- (3) Grasp results exactly from thinking mathematically or mathematical problem solving
- (4) Two adequacy for posing problem
 - to be good for aims of lesson
 - to lead results to be good for problem posed
- (5) Problem posing and the result of solving the problem posed
 - to consider the characteristic of problem; self-creating aspect
- (6) Developing the mind of challenge, confidence, feeling of effectiveness
- (7) Collaboration and creating
 - Japanese proverb; sann-ninn yore-ba monnjyu –no chie

How to develop Student's Mathematical Thinking in Classroom

Objective: 1) To find and think about average 2) To develop mathematical thinking and children's image from surroundings.

Field note :

13:24 ~

T This is Japan's map. Around country is sea, I will give this to your teacher. Also these color papers. I will give you a card with Japan's view.



13:30

T. I will stick this paper. Watch it carefully.

What it is?

C. Calendar

S. How to say Sunday?

C. Sunday...Saturday

T. I will hide somewhere.

C. It looks like Hospital sign.

S	m	t	w	th	f	s
			1	2		4
5	6	7	8			
12	13	14	15	16		18
19	20	21	22	23	24	25
26	27	28	29	30	31	

T. Summary of Blue is bigger or Red is bigger?

Blue $3+7+14$ Red $6+7+8$

Almost C. answer Red is bigger than Blue

T. Why do you think so?

C. Cause it has 9 so it's bigger

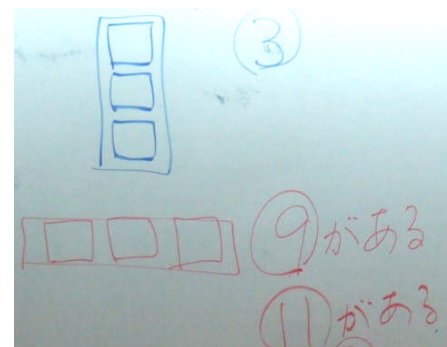
T. It has 3 numbers

T. I will ask you again. Summary of Blue is bigger or Red is bigger?

Nobody answered Blue is bigger

C. Because Blue has 3, but red has 11

T. We will check it by calculate them



T. Had already learnt addition right?

T. Blue is $3+10+17$ right?

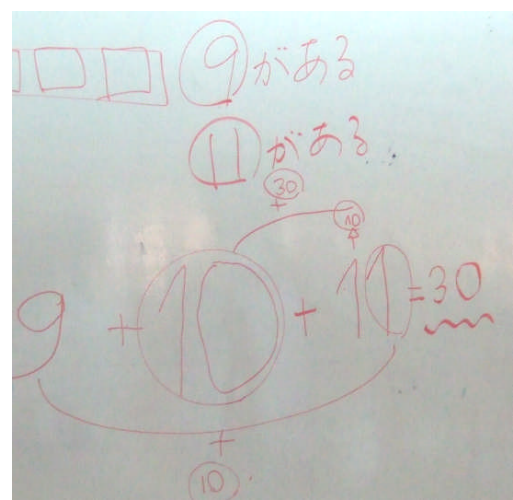
T. Red is $3+10+17$ right?

C. Wrong!!

T. Can you correct it?

C. $9+10+11$

T. What is the answer?

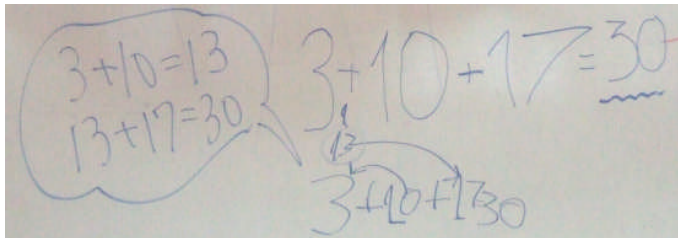


- C. Blue 30. Clap. Red 30. clap
- T. Blue is 30. Red is 30. They are same.
- T. How do you calculate the red one?
- C. $9+11$ is 20 then $20 + 10$ is 30
- T. Can you draw the lines?
- T. Look it carefully. Do you have other way to calculate?
- C. $3+10+17$. $10+3$ is 13 then $13+17$ is 30

13:52

T. She/he calculates from left to right. But another he/she did the faster way to calculate. **It is easier to calculate, when we have three of tens. Can we do like this in that one.**

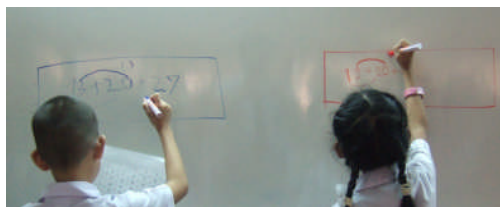
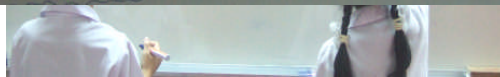
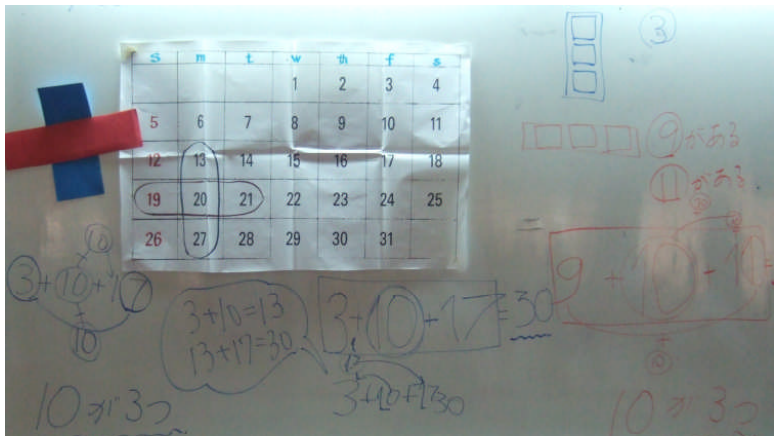
- T. discuss with your friend
- C. Draw the line 3 and 7 of 17
- T. 10, 10, and 10 are 30
- Both have there of tens



We can calculate faster if make there of tens.
10 is central, inside the card.
 If we can find this middle number, we can calculate faster.

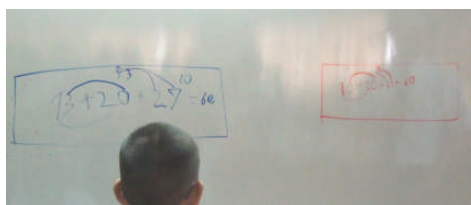
- T. I'll change cards position.
- What are summary of them?
- Blue: $13+20+27$ a lot
- Red: $19+20+21$ a little
- Same: a little

- T. Please. Calculate them.
- C. Same
- T. What it is?
- C. 60
- T. Writes the equation please.



14.02

T. It is really to be 60?



14.07

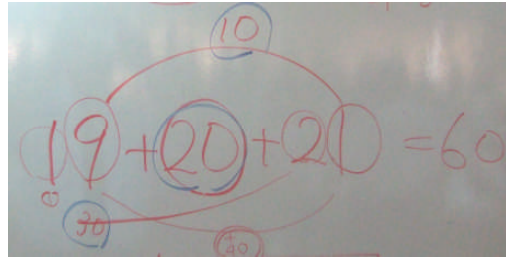
T. Is it possible to leave the central number?

C. Yes. $19+20+21$, $19+1$ is 20.
 $20+20+20$

T. What about blue one.

C. $13+20+27$, $3+7$ is 10. $20+10$ is 30

T. Good Job! It faster to calculate when leaves central number.



14.12

T. If on the calendar, wherever we put cards it will be same?

T. Anywhere you want to put cards?

C. $16+23+30$

C 赤 :

T 同じになる: 違う: 分から

ない:

T. I will take cards off and circle around, instead.

