Findings for Mathematics

The purpose of Achieve's analysis was to determine similarities and differences among the 12 participating APEC member economies' expectations and priorities in three main areas:

(A) Qualitative aspects of standards (determining whether the standards are organized into single grade levels or multiple grade bands, how the standards are organized in regard to key strands and broad topics, and what level of detail they contain);

(B) Core content expectations (describing what topics economies address in their standards, the extent to which those topics are common across most or all of the economies, and the proportion of economies standards that is made up of the common topics); and

(C) Performance expectations (describing the emphasis given to various levels of performance skills – procedural skills, conceptual understanding skills, strategic problem solving and reasoning skills – evident in the member economies' standards).

Twelve APEC economies volunteered for participation in the mathematics portion of the study: Australia, Canada, China, Chinese Taipei, Hong Kong, Japan, Korea, Malaysia, New Zealand, Singapore, Thailand and the United States. (Some economies that otherwise would have chosen to participate could not because an English translation of the standards for comparison was required for the analysis.) The complete list of standards coded for this study is available in Appendix A.

Overall, Achieve observed great variety from economy to economy in terms of the organizational aspects of the standards, including how the member economies organize their mathematics content standards in regard to key strands and broad topics. Despite these differences, Achieve also found a *common set of topics* that is addressed by the majority of economies at each grade span, particularly at the primary grades, indicating a level of international agreement about what mathematics is most important for students to learn. Finally, Achieve determined that the level of performance, or cognitive skill, expected across the participating economies was generally at the lower end of the performance continuum.

What follows are the detailed findings of Achieve's analysis.

A. QUALITATIVE ASPECTS OF THE STANDARDS

To understand the similarities and differences in the way standards are crafted across the economies, Achieve examined how the standards are structured (grade by grade vs. grade spans) and what mathematical strands and topics they include. Despite the considerable variety in structure, standards from the 12 participating economies are more similar than they are different – particularly at the elementary grades.

Grade Level v. Grade Span Approach

There are two approaches to organizing standards: by grade level (i.e., listing objectives and standards for each grade) and by grade spans (i.e., listing objectives and standards over a period

of two, three or four years). The majority of APEC economies studied organize their mathematics standards by grade level: Of the twelve economies, five use the single grade approach for all standards, four blend single grade and multiple-grade groupings, and three use grade spans.

TABLE 4: Organization of Mathematics Standards: Single Grades v. Grade Spans

Economies	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
Australia		✓		✓		✓		✓				
Canada	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
China		✓			✓			✓		✓	✓	
Chinese Taipei	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Hong Kong	✓	✓	✓	✓	✓	✓		✓			✓	
Japan	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Korea	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		
Malaysia	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
New Zealand[1]		✓		✓		✓		✓		✓	✓	✓
Singapore	✓	✓	✓	✓	✓	✓	✓	✓	✓			
Thailand		✓			✓			✓			✓	
United States[2]			✓			✓			✓			

[1] *New Zealand presents its "blended" mathematics standards in overlapping bands in recognition of the varying pace at which students master material. For the purpose of this report, each set of standards are placed in the chart above in the grade or grade span in which the heaviest emphasis on that set of skills takes place and the grade or grade span in which a majority of students will likely master those skills.*

[2] *The United States has not established national standards (there are 50 different sets of state standards), but Achieve included the National Assessment of Educational Progress (NAEP) assessment framework for mathematics (2007) in this study.*

Years of Required Mathematics Instruction

Achieve surveyed economies to determine how many years of total instruction is required for students. We found that, while all economies begin mathematics instruction at grade 1, if not earlier in the kindergarten year, there is some variation in the total number of years of mathematics required across economies. Seven of the 12 APEC economies in this study have established requirements for the minimum number of years of mathematics instruction, ranging from nine to 11 years, with an average of 10 years. [Note: the number of years of required mathematics does not necessarily correspond to the number of years covered by the standards.] In four economies, mathematics requirements are established "locally," such as at the province, territory or state level. The table below shows the number of years of mathematics required across economies.

TABLE 5: Years of Mathematics Instruction Required by Economies

Economy	Years of Required Mathematics Instruction
Chinese Taipei	11
Hong Kong	9
Japan	11
Korea	10
Malaysia	10
Singapore	10
Thailand	9
Australia	Requirements vary, depending on the state or territory
Canada	Requirements vary, depending on the province or territory
New Zealand	Requirements vary, depending on the locality
United States[1]	Requirements vary, depending on the state
China	Not available

[1] In the United States, all but four states have set statewide graduation requirements. Of the remaining states, 18 require four years of high school mathematics, 22 require threes years and the remaining six require two years. For more information about state graduation requirements, visit http://www.achieve.org/GradRequirements.

Strand Organization & Emphasis by Grade Span

The 12 economies in this analysis organize their standards according to a variety of strands, or domains, using varying levels of specificity; some are very broad categories, while some use more specific topics to arrange their standards. Many economies use similar strand titles to organize their standards.

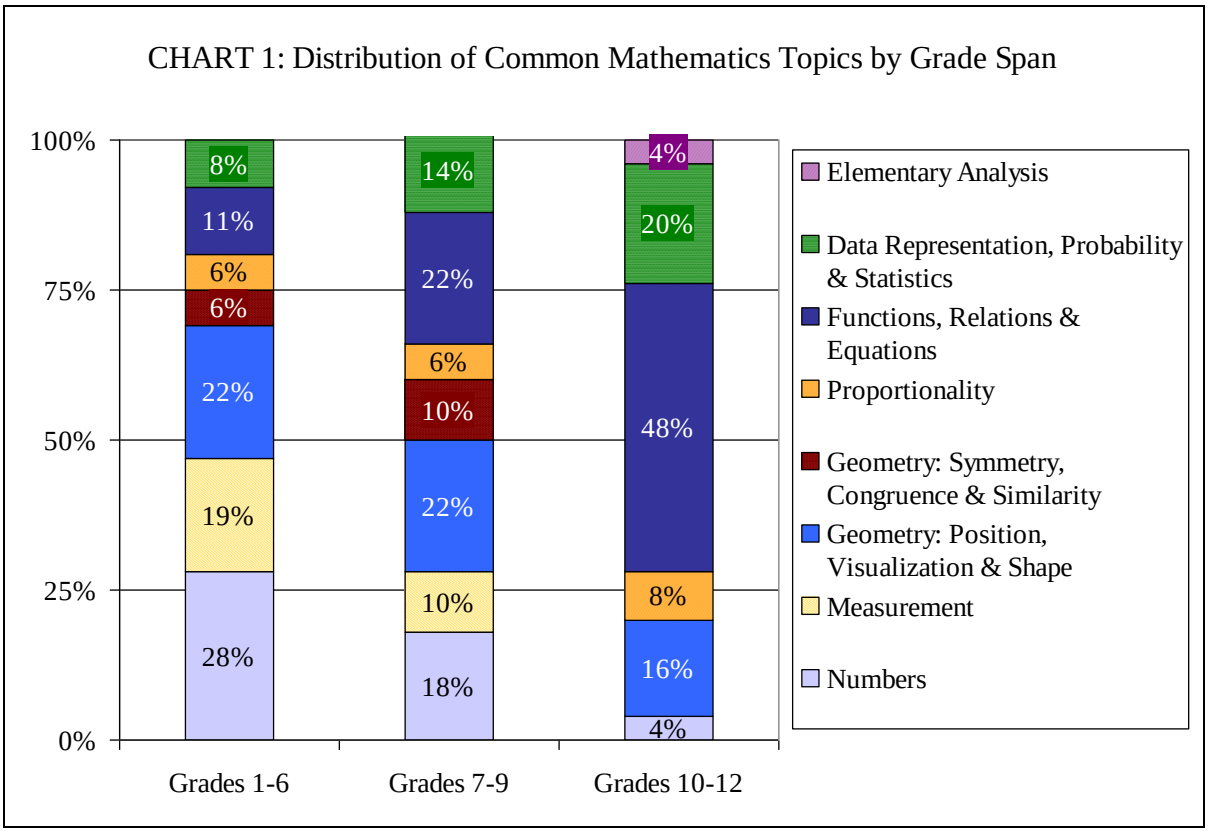
To categorize the economies' mathematics standards, Achieve used a mathematics coding framework developed by Michigan State University that organizes content into 10 strands:

1. Number
2. Measurement
3. Geometry: Position, Visualization & Shape
4. Geometry: Symmetry, Congruence & Similarity
5. Proportionality
6. Functions, Relations & Equations
7. Data Representation, Probability & Statistics
8. Elementary Analysis
9. Validation & Structure
10. Other Content

Achieve noted that the emphasis on certain domains of mathematics shifts clearly across the grade spans, as demonstrated in the graph below. In grades 1-6, collectively across all economies, Number, Measurement and Geometry (position, visualization and shape) compose on average nearly 70 percent of economies standards. This diminishes 50 percent in grades 7-9 and to 20 percent in grades 10-12.

In contrast, the emphasis on Functions, Relations and Equations, relative to other strands across all economies, increases with the grade levels, totaling 11 percent in grades 1-6, and increasing to 22 percent in grades 7-9, and to 48 percent in grades 10-12. Similarly, the emphasis on Data increases over the grades (albeit to a lesser degree), from eight to 14 to 20 percent. The upper secondary span, grades 10-12, includes attention to Elementary Analysis topics, which include, among other content, topics typically addressed in pre-calculus and/or calculus courses.

These inverse shifts in emphasis suggest that economies tend to emphasize number sense and number operations in the early grades, which beyond teaching students vital skills and concepts, lays the groundwork for the study of more abstract concepts presented in the domain of algebra later in a student's career.



Level of Detail

The economies' mathematics standards vary in respect to their level of detail. As the table below indicates, some economies use descriptive language and/or provide elaborations or examples in their mathematics standards. Others use sparer, shorter statements to convey what students need to know. Because Achieve coded English-language versions of all standards, some style differences in the language may be due to the challenges of translation. Yet even within the group of standards written originally and only in English (Australia, Canada, New Zealand and United States), the economies include varying levels of detail in their standards. This, coupled with the organization and structure of economies' standards, results in widely different lengths of the standards documents across economies. In addition, some economies provided only summaries of their standards.

TABLE 6: Level of Detail: Illustrative Examples

Topic	Korea	Canada
Collecting data (Primary Level)	**Grade 3:** By collecting, sorting, and organizing various data, express them in tables, bar graphs, and simple pictographs.	**Grade 2:** Gather and record data about self and others to answer questions. Formulate a question that can be answered by gathering information about self and others; Organize data as it is collected using concrete objects, tallies, checkmarks, charts or lists; Answer questions using collected data.
Pythagorean Theorem (Lower Secondary Level)	**Grade 9:** Understand and prove Pythagorean theorem. Apply Pythagorean theorem to simple figures.	**Grade 8:** Develop and apply the Pythagorean theorem to solve problems. Model and explain the Pythagorean theorem concretely, pictorially or using technology; Explain, using examples, that the Pythagorean theorem applies only to right triangles… (continues)
Quadratic Equations (Upper Secondary Level)	**Grade 10:** Understand the meanings of real root and imaginary root of a quadratic equation. Understand the discriminant of a quadratic equation. Understand the relation between the root and coefficient of a quadratic equation.	**Grade 10:** Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using factoring, the quadratic formula, and graphing. Determine the character of the real and non-real roots of a quadratic equation, using the discriminant in the quadratic formula and graphing.

B. CORE CONTENT EXPECTATIONS

Achieve set out to examine the core content included in the standards across the different economies to determine the extent to which there is commonality. To do this, we analyzed the topics treated by each economy at each grade level or span. Although there is variation across economies, Achieve found that there are a number of topics that are common across economies. For the sake of reporting, Achieve focused on three grade level spans: primary (grades 1-6); lower secondary (grades 7-9); and upper secondary (grades 10-12). Although the standards may be written in varying degrees of detail and may be introduced and emphasized at different grade levels, there is an identifiable set of common topics across most or all of the economies participating in this study at each grade span.

Common Topics across Economies by Grade Span

Achieve's analysis found that there is a set of topics at each grade span that are common across the economies. The decision rule for inclusion is based on a constant percentage: 67 percent or more of economies included in any given grade span must address the topic in order for it to be considered a shared or common topic. (Note: The number of economies included in each grade span varies, as not all economies have standards at every grade level; therefore, the number of economies required for a topic to be included in the list is different from span to span.) The

topics included at each grade span are listed below alongside the percentage of economies addressing that topic in their standards. The topics are organized by the categories in the coding framework (Number, Measurement, etc.).

TABLE 7: Common Topics across Economies by Grade Span

	GRADES 1-6 (12 ECONOMIES) 100%=12 Economies 92%=11 Economies 83%=10 Economies 75%=9 Economies 67%=8 Economies	GRADES 7-9 (12 ECONOMIES) 100%=12 Economies 92%=11 Economies 83%=10 Economies 75%=9 Economies 67%=8 Economies	GRADES 10-12 (11 ECONOMIES) 100%=11 Economies 91%=10 Economies 82%=9 Economies 73%=8 Economies
TOPICS	**GRADES 1-6 % of Economies**	**GRADES 7-9 % of Economies**	**GRADES 10-12 % of Economies**
NUMBER*			
Meaning	100%		
Operations	100%		
Properties of Operations	67%		
Common Fractions	100%		
Decimal Fractions	100%		
Relationships of Common & Decimal Fractions	83%	67%	
Percentages	83%	75%	
Negative Numbers, Integers & Their Properties		100%	
Rational Numbers & Their Properties		67%	
Real Numbers, Their Subsets & Properties		92%	73%
Exponents, Roots & Radicals		92%	
Number Theory	100%	75%	
Rounding & Significant Figures	67%	92%	
Estimating Computations	92%	67%	
MEASUREMENT			
Concept of measure (including non-standard units)	100%		
Standard units (including metric system)	100%		
Common measures	100%	75%	
Computations, formulas and properties of length and perimeter	92%	92%	
Computations, formulas and properties of area	100%	92%	
Computations, formulas and properties of surface area		92%	
Computations, formulas and properties of volumes	83%	92%	
Estimation of measurement and	92%		

TOPICS	GRADES 1-6 % of Economies	GRADES 7-9 % of Economies	GRADES 10-12 % of Economies
errors of measurement			
GEOMETRY: POSITION, VISUALIZATION & SHAPE			
Line and coordinate graphs		92%	91%
Equations of lines in a plane			82%
Points, lines, segments, half-lines, and rays	75%	92%	82%
Angles	83%	92%	
Parallelism and perpendicularity	75%	92%	
2-D Geometry: Polygons & Circles		67%	
Triangles and quadrilaterals: their classification and properties	100%	100%	
Pythagorean Theorem and its applications		100%	
Other polygons and their properties	100%	100%	
Circles and their properties	100%	92%	82%
3-Dimensional shapes and surfaces and their properties	100%	100%	
Spatial perception and visualization	83%	83%	
GEOMETRY: SYMMETRY, CONGRUENCE & SIMILARITY			
Patterns, tessellations, friezes, stencils, etc.	67%		
Symmetry	75%	75%	
Transformations		83%	
Congruence		83%	
Similarities (similar triangles and their properties; other similar figures and properties)		92%	
Constructions w/ Straightedge & Compass		83%	
PROPORTIONALITY			
Meaning of ratio and proportion	67%	92%	
Solving practical problems with proportionality		83%	
Scales (maps and plans)	67%		
Proportion based on similarity		67%	
Slope and gradient in straight line graphs			82%
Trigonometry of right triangles			100%
FUNCTIONS, RELATIONS, & EQUATIONS			
Number patterns	83%	83%	
Functions and their properties		67%	91%
Representation of relations and functions		83%	91%

TOPICS	GRADES 1-6 % of Economies	GRADES 7-9 % of Economies	GRADES 10-12 % of Economies
Relationship of functions and equations			73%
Interpretation of function graphs		83%	82%
Linear Functions		92%	
Quadratic Functions			82%
Trigonometric Functions			91%
Representation of numerical situations by equations	92%	92%	
Evaluating expressions		75%	
Equivalent expressions (including factorization and simplification)		83%	82%
Linear equations and their formal (closed) solutions		100%	91%
Quadratic equations and their formal (closed) solutions			100%
Polynomial equations and their solutions			73%
Inequalities and[/or] their graphical representation		67%	82%
Systems of equations and their solutions (including matrix solutions)		75%	82%
Substituting into or rearranging formulas	67%	67%	73%
DATA REPRESENTATION, PROBABILITY, & STATISTICS			
Collecting data from experiments and simple surveys	92%	83%	
Representing data	100%	92%	82%
Interpreting tables, charts, plots, graphs	100%	92%	82%
Measures of central tendency	67%	92%	
Measures of dispersion, variance			82%
Use and misuse of statistics		75%	
Informal likelihoods and the vocabulary of likelihoods		92%	
Numerical probability and probability models		92%	82%
Counting principles			73%
ELEMENTARY ANALYSIS			
Arithmetic and geometric sequences			73%
	Total Topics = 36	**Total Topics = 52**	**Total Topics = 26**

** Two categories – Validation and Structure and Other Content – do not contain any topics that meet the 67% or more of economies threshold to be considered a shared or common topic.*

Primary School: Grades 1-6

The primary school standards introduce essential basic concepts and skills. Within the Number strand, most economies expect students in grades 1-6 to learn number sense and operations with whole numbers, fractions, decimals and percents, as well as how to order and compare such numbers. In addition to the basic operations (addition, subtraction, multiplication and division), standards at this level address basic number theory concepts.

With regard to the Measurement strand, economies expect primary students to learn common measures (such as length, time and temperature), units, conversion between units and estimation of measurements. The standards also tend to cover perimeter, area, volume and calculation of these measurements for a variety of shapes and figures. In Geometry, the set of common topics includes the properties and classification of two- and three-dimensional shapes, facilitated by study of angles, parallelism and perpendicularity. Students also learn about proportionality, particularly by reading and interpreting maps. Not only do economies expose students to basic transformational geometry concepts, such as symmetry and patterns, but also they promote spatial perception[3] with respect to geometric figures and shapes.

A Closer Look at Data in the Primary Grades

All economies expect students in the elementary grades to be able to represent data in tabular or graphic form and to interpret that data.[i]

67 percent of economies also expect these students to be able to calculate measures of central tendency such as mean, median and mode, using data.[ii]

Just two economies expect students at this level to be able to go a step beyond interpretation to use data to make predictions.[iii]

Very few algebra concepts are included in the set of common topics at this level. However, patterns are included in connection with content in the Number and Geometry strands. The remaining algebra concepts lay the foundation for more advanced content students will encounter in secondary school: understanding the use of variables, expressions and equations in abstract representation and substituting values into formulas.

Data is included at this level, specifically basic data collection (e.g., simple surveys) and the representation and interpretation of that data in a variety of formats, including tables, charts and graphs. Economies also commonly address measures of central tendency.

Lower Secondary School: Grades 7-9

Economies share more topics in common at this level than in the primary and upper secondary school levels; there are 44 percent more common topics at this level than the primary school

[3] Students are able to gain a sense of spatial perception conceptually by learning that to move from a two-dimensional figure (for example, a square) to a three-dimensional figure (for example, a cube), it is necessary to introduce the element of height. The area of a square of side s is $A=s \times s = s^2$ while the volume of a cube is $V= s \times s \times s = s^3$. In addition, students also learn to visualize solids and surfaces in three-dimensional space when given two-dimensional representations (such as nets or multiple views) and to create two-dimensional representations for the surfaces of three-dimensional objects.

level and twice as many topics in common as in upper secondary school standards. Across most strands, the common topics build on concepts from grades 1-6. For instance, the Number topics expand beyond whole numbers, fractions, decimals and percents to include treatment of integers, both conceptually and operationally. In addition, economies commonly call for students to conduct multi-step operational problems with rational numbers, requiring the application of order of operations and absolute value. More sophisticated and abstract thinking play a larger role at this juncture; students across these economies are generally expected to understand integer exponents and their properties and the relationship between roots, radicals and rational exponents. They are expected to round numbers and work with significant digits.[4]

At this level, many economies include line and coordinate geometry, the Pythagorean Theorem and its applications, transformations and congruence in the Geometry strand. Students are generally expected to solve problems by applying concepts of proportionality and similarity, as well as to apply concepts of symmetry, congruence and similarity to perform geometric constructions. While economies at the lower secondary school level generally continue to address such measurement topics as perimeter, area and volume that were also addressed at the primary school level, they expand their treatment of three-dimensional geometry to include the computation of surface area. Proportionality is further developed in the lower secondary grades, covering not only the meaning but also the use of proportionality in solving practical problems.

As the emphasis on algebra grows across the grade spans, the set of common topics expands to include functional relationships and their graphs, with a focus on linear functions. Students in these economies are generally expected to be able to solve linear equations and their systems. Simplification and factorization, skills essential to solving such equations, are included at this level also. The primary grade skill of substituting into formulas extends into the evaluation of algebraic expressions and rearrangement of formulas. Economies also tend to cover inequalities and their graphical representations.

The overlapping topics in the Data strand reflect a greater degree of sophistication in the types of plots and graphs students must construct and interpret. The shared content at this level extends beyond basic summary statistics – including the

A Closer Look at Algebra in the Lower Secondary Grades

All economies at the lower secondary level expect students to have familiarity with linear equations and to be able to solve them.[iv]

75 percent of economies expect these students to do the more demanding task of working with systems of linear equations.[v]

Just two economies expect students at this level to work with families of functions, including the effect on graphs of functions when the coefficients of the equation change.[vi]

[4] In applications of numbers in the sciences or financial disciplines, the need often arises to maintain consistency with respect to the level of precision in the data and in the answer calculated from the data. Typically, when multiplying or dividing, the answer should have the same number of significant figures as the data with the smallest number of significant digits. When adding or subtracting, the answer should have the same number of decimal places as the data with the smallest number of decimal places. Significant figures are often associated with rounding – particularly when rounding of an answer is a primary contributor to its uncertainty.

calculation of central tendency – to cover the uses and misuses of statistics, as well as basic concepts in probability.

Upper Secondary School: Grades 10-12
The standards analyzed at this level include content from both required courses and any optional courses taken by more than 50 percent of students – courses in the common pathway. There are the fewest shared topics at this level – half the common topics in the lower secondary level. Some topics from the previous grade span are revisited at this level. For instance, although the number of Geometry topics in common decrease at this level, the few topics carried over – line and coordinate graphs, circle properties and basic two-dimensional concepts – are joined by the expectation that students be able to understand equations of lines in a plane. Right triangle trigonometry and slope in line graphs also appear in this grade span, linked with the addition of trigonometric functions in algebra.

In algebra, the common topics demonstrate that economies tend to build on the grades 7-9 content and place emphasis on the relationship between functions and equations. The algebra content moves beyond linear functions to address non-linear functions, specifically quadratic and trigonometric functions and the solution of quadratic and polynomial equations. Consequently, the determination of equivalent expressions expands to include the factoring and simplification inherent in solving more advanced equations.

The topics economies emphasize in the Data strand suggest a tendency in these economies to maintain a focus on data representation and interpretation, but to apply it to more sophisticated types of plots and graphs. Upper secondary school standards expand on measures of central tendency (covered in the two preceding grade spans) to cover measures of dispersion of data. Finally, the data topics here include counting principles, such as permutations and combinations.

A Closer Look at Algebra in the Upper Secondary Grades

All economies expect students at the upper secondary level to have familiarity with quadratic equations and their solutions.[vii]

73 percent of economies expect students to work with polynomial equations, a more sophisticated type of equation often requiring more than the rote mechanisms used to solve quadratic equations.[viii]

Just two economies expect students to be able to solve parametric equations, which tend to involve multiple variables. These equations are typically taught in calculus courses.[ix]

This is also the only level at which Elementary Analysis receives any attention, and only briefly with the inclusion of arithmetic and geometric sequences. This is not unexpected given the advanced nature of this content.

Summary of Common Topics
Achieve's analysis indicates that at the Primary School level, there is a robust set of common topics that includes an emphasis on Number Sense, Number Operations and Measurement, which provide students with foundational knowledge and skills they need to be successful in other domains of mathematics, such as algebra. Measurement and geometry concepts – including

measures, units, perimeter, area, volume and a basic understanding of two-dimensional figures – provide foundational knowledge and skills that students can then apply in more sophisticated and abstract contexts later in their schooling. The few algebra and data concepts in the common topics at the primary level serve as foundations upon which greater sophistication is built at the lower and upper secondary levels.

By the upper secondary level, the set of common topics has decreased. Collectively, only seven common topics are noted across the strands of number measurement and geometry. Thirteen common topics are noted for algebra/functions and five for Data Representation, Probability and Statistics, reflecting less commonality across economies, likely as a result of a greater number of curricular choices for students.

Featured Economy: New Zealand's focus on Data Representation
Unlike most countries, New Zealand devotes about a third of its standards to statistics at every grade span. Each pass through statistical content emphasizes the *statistical enquiry cycle*, placing individual tasks and skills in the context of a larger process of research and discovery. By 5th grade[5], the standards indicate that students are "gathering, sorting, and displaying multivariate category data, discrete numeric data and simple time-series data to answer questions." At 10th grade,[6] students are planning and conducting their own surveys and experiments. By the end of secondary school[7], they have critiqued and refined the process of statistical enquiry using margins of error, experimental randomization schemes, data modeling and more. These expectations are considerably different from other economies' expectations of their students, not only in the level of mastery expected, but in the consistent focus across all grade levels on data.

Topics that persist across grades spans
Achieve found that some of the common topics are covered across economies in more than one grade span. The table below shows the common topics, as well as the percent of economies that address those topics in each grade span.

[5] See http://nzcurriculum.tki.org.nz/the_new_zealand_curriculum/learning_areas/mathematics_and_statistics/mathematics_and_statistics_curriculum_achievement_objectives#level%203
[6] See http://nzcurriculum.tki.org.nz/the_new_zealand_curriculum/learning_areas/mathematics_and_statistics/mathematics_and_statistics_curriculum_achievement_objectives#level%208
[7] See http://www.nzqa.govt.nz/ncea/assessment/search.do?query=Statistics&view=all&level=03#achievements

TABLE 8: Mathematics Topics that Persist across Grades Spans

MAJOR MATHEMATICS AREAS Sub-topics	Grade Span 1-6 (12 economies)	Grade Span 7-9 (12 economies)	Grade Span 10-12 (11 economies)
GEOMETRY: POSITION, VISUALIZATION & SHAPE			
Points, lines, segments, half-lines, & rays	75% (9/12)	92% (11/12)	82% (9/11)
Circles & their properties	100% (12/12)	92% (11/12)	82% (9/11)
FUNCTIONS, RELATIONS, & EQUATIONS			
Substituting into or rearranging formulas	67% (8/12)	67% (8/12)	73% (8/11)
DATA REPRESENTATION			
Representing data	100% (12/12)	92% (11/12)	82% (9/11)
Interpreting tables, charts, plots, graphs	100% (12/12)	92% (11/12)	82% (9/11)

The fact that some topics are included across multiple grade spans raised questions for the Achieve analysts about whether standards are redundant from grade to grade. A closer look, however, shows that while topics may be repeated, their coverage increases in depth and challenge as the grade spans advance. TABLE 9 illustrates how two topics from TABLE 8 – "representing data" and "circles and their properties" –increase in complexity (albeit with some differences) in the standards of four economies: Alberta, Canada; Chinese Taipei; Korea and Singapore.

TABLE 9: Illustrations of Mathematics Topics that Persist across Grades Spans

Grade Spans	Alberta, Canada	Chinese Taipei	Korea	Singapore
REPRESENTING DATA				
Primary	**Grade 3:** Collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions.	**Grade 1:** Students can classify and record simple events and activities in daily life.	**Grade 2:** By using simple pictures, express investigated data in graphs and compare the size of data.	**Grade 3:** reading and interpreting bar graphs in both horizontal and vertical forms, reading scales, and completing a bar graph from given data.
	Grade 6: Create, label and interpret line graphs to draw conclusions.	**Grade 4:** Students can report statistical charts of data in daily life, such as bar chart, line chart and pie chart.	**Grade 5:** Organize data, express them in Stem and leaf diagram, or pictographs, and grasp the properties of the data.	
Lower Secondary	**Grade 7:** Construct, label and interpret circle graphs to solve problems.	**Grade 9:** Able to organize raw data into simple tables and statistical graphs to represent the hidden meaning of data.	**Grade 7:** Understand the distribution of relative frequency and cumulative frequency, and know how to make a graph of it.	**Grade 7:** construction and interpretation of tables, bar graphs, pictograms, line graphs, pie charts, histograms
Upper Secondary	**Grade 10:** Represent data using function models.	**Grade 11:** Understand the data in charts, data showing centralized tendency, data showing dispersion tendency and integrate centralized tendency and dispersion tendency.	This topic is not addressed in Korea's grade 10 standards; no other standard sets for other upper secondary grades were available.	**Grade 10:** drawing graphs from given data [related to] problems derived from practical situations such as… simple interest and compound interest, money exchange, profit and loss, taxation.
CIRCLES & THEIR PROPERTIES				

Grade Spans	Alberta, Canada	Chinese Taipei	Korea	Singapore
Primary	**Grade 2:** Describe, compare and construct 2-D shapes, including triangles, squares, rectangles, circles.	**Grade 3:** Students can draw circles with compasses and recognize the center of a circle, its circumference, radius and diameter.	**Grade 2:** Understand segments, straight lines, triangles, quadrangles, and circles, and know how to make or draw these shapes.	**Grade 2:** Forming, square, triangle, semicircle, and quarter circle.
		Grade 6: Comprehend the formulas of area and perimeter of a circle and apply this knowledge to find the area of circular sectors.	**Grade 6:** Understand the method of calculating the circumference and the area of a circle, and calculate them.	**Grade 6:** Finding the area and perimeter of a figure made up of some of the following shapes: square, rectangle, triangle, semicircle and quarter circle.
Lower Secondary	**Grade 7:** Demonstrate an understanding of circles by describing the relationships among radius, diameter and circumference.	**Grade 8:** Able to recognize geometrical attributes and related terms of circles (center, radius, hypotenuse, diameter, arc, segment, central angle, and sector).	**Grade 9:** Understand the properties of a chord in a circle; Understand the properties of a circle's tangent line.	This topic is not addressed in the Singapore lower secondary standards.
Upper Secondary	**Grade 11:** Solve problems using a variety of circle properties and relevant trigonometric ratios, and justify the solution strategy used.	This topic is not addressed in the Chinese Taipei upper secondary standards.	**Grade 10:** Find the equation of a circle; Understand the positional relation of a circle and a line on the coordinate plane.	**Grade 10:** Symmetry and angle properties of circles (e.g., tangents from an external point are equal in length, angles in opposite segments are supplementary).

As TABLE 9 shows, each economy emphasizes collecting data on issues that pertain to daily life in the standards for the early grades. While the standards vary in terms of the specifics and pacing of the graph types that students are expected to learn over time, all four economies expect their students to master a collection of graph types including line, bar, pie and histogram by the end of the lower secondary grades and to continue toward more challenging explorations of data in the upper secondary grades.[8]

Similarly, the standards covering the topic of circles and their properties show a pattern of both diverse content and increased depth across the grade spans. Where students in Singapore begin in grade 2 by forming shapes from cut-outs, four years later they are finding the area and perimeter of composite figures, and by grade 10 they are exploring the symmetry and angle properties of circles. In Alberta, Canada, there is emphasis on shape classification in the early grades, describing relationships among radius, diameter and circumference in grade 7, and applying trigonometric ratios to the geometry of the circle in grade 11. Each approach offers a slightly different template for guiding students from the fundamentals through the finer points of an important topic as they move from childhood toward the adult world. In each case, however, topics covered deepen in complexity over the grades.

[8] Although the available upper secondary standards from Korea do not include specific coverage of the Representing data topic, they do clearly touch on data as a subject of study. Also worth noting is that translations of standards for grades 11 and 12 in Korea were not available at the time of this report.

Common Topics as Proportion of Economy Standards at Different Grade Spans

Having identified a set of topics that most economies address in common, Achieve was then able to determine what proportion of the content addressed in each economy's standards is comprised of that set of topics. Said differently, Achieve was able to quantify the extent to which the economies' standards are focused around the common set of topics, or whether they include a lot of additional content as well.

TABLE 10 below indicates that at grades 1-6, on average, 76 percent of the content topics addressed across the 12 economies are those included in the set of common topics; 24 percent of the topics across the economies at that grade span are outside of the common set of topics. In grades 7-9, an average of 68 percent of the content included in the standards across the economies is found in the set of common topics. Finally, at the upper secondary level, only an average of 34 percent of the content included in the standards across the economies are found in the set of common topics.

TABLE 10: Overall Overlap between Standards & Common Set of Topics

Grade 1-6 Average	Grade 7-9 Average	Grade 10-12 Average
76% Range: 71%-89%	**68%** Range: 59%-85%	**34%** Range: 26%-55%

These data reflect the fact that on average the proportion of the content addressed by the economies in their standards that are from the set of common topics decreases as the grade levels progress. Looking more specifically at the upper secondary level, as TABLE 11 shows, this trend continues: On average, the set of common topics comprises 47 percent of the content for economies' standards that extend only through grade 10, while they comprise only 32 percent of the content for economies whose standards extend through grade 12.

TABLE 11: Proportion of Content Addressed in Economies' Standards From Common Topics: Grades 10, 11 and 12

Economy	Percent overlap	Final grade of standards coded
Korea	55%	10
Singapore	40%	10
Average: Economies where the common pathway ends at grade 10	**47%**	
China	31%	11
Japan	28%	11
Malaysia	33%	11
Average: Economies where the common pathway ends at grade 11	**31%**	
Alberta, Canada	32%	12
Chinese Taipei	30%	12
Hong Kong	35%	12
New Zealand	26%	12
Thailand	41%	12

Economy	Percent overlap	Final grade of standards coded
United States	26%	12
Average: Economies where the common pathway ends at grade 12	**32%**	
Overall Average	**34%**	

C. PERFORMANCE EXPECTATIONS

In addition to the content, Achieve also analyzed the performance, or cognitive skill expectations of the standards from the 12 economies in this study. Our goal was to determine the balance of basic skills, such as recall, and advanced skills, such as applying advanced mathematical reasoning, across all economies. Below is a direct accounting of the skill expectations contained in the economies' standards that addresses the question of balance.[9]

Mathematics Performance Categories & Levels

Performance expectations from the coding taxonomy have been grouped into a hierarchy of levels approximating increasing levels of cognitive demand. The levels, in increasing order of cognitive demand, are:

1. Recall
2. Using routine procedures and tools to solve problems
3. Using more complex procedures and conceptual understanding to solve problems
4. Formulating problems and strategizing/critiquing solution methods
5. Applying advanced reasoning skills

Level 1 includes demonstrating basic knowledge or recall of a fact or property. Level 2 includes routine problem solving that asks students to do such things as compute, graph, measure or apply a mathematical transformation. Level 3 includes estimating, comparing, classifying and using data to answer a question, or requiring students to make decisions that go beyond a routine problem-solving activity. Level 4 includes asking students to formulate a problem or to strategize or critique a solution method. Level 5 includes asking students to develop algorithms, generalizations, conjectures, justifications or proofs.

The listing of skills included in each performance category is included in Appendix H.

Emphasis of Performance or Cognitive Skills across Grade Spans

Achieve found that most of the economies place the greatest emphasis on the Level 1 and 2 performance expectations – i.e., Recall and Using Routine Procedures – and less emphasis on Level 5, Applying Advanced Reasoning skills. The heavy emphasis on the Level 1 skills – those that come under the Recall category – is due partly to the fact that economies often address both recall skills and more advanced skills in a single standard. Skills found in the Recall category are often necessary – and articulated in the standards – to lay the foundation for students to be able

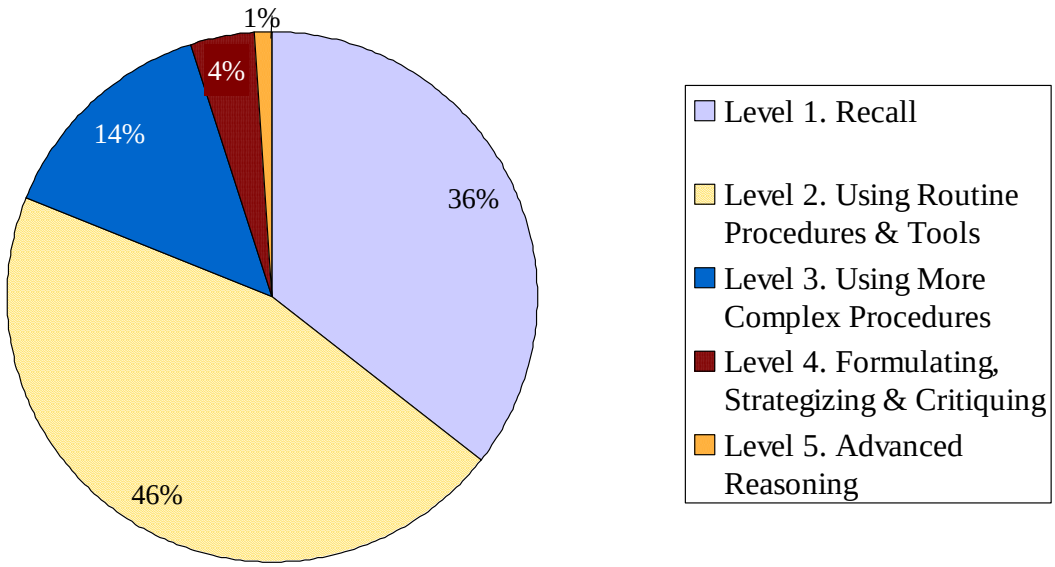
[9] To look beyond balance of performance expectations and compare the rigor of economies' standards was not the intent or purpose of this study. To answer questions regarding comparative rigor of expectations would require additional analysis.

to apply higher-order skills with the very same content. In cases where a standard addressed more than one skill in a standard, Achieve analysts recorded both skills in its analysis.

Primary School: Grades 1-6[x]

Over 80 percent of performances expected of students across the economies in grades 1-6 consist of Level 1 and 2 skills – Recall and Using Routine Procedures. Less than 20 percent of the performances described in the standards address more cognitively demanding skills, with only five percent addressing Levels 4 and 5.

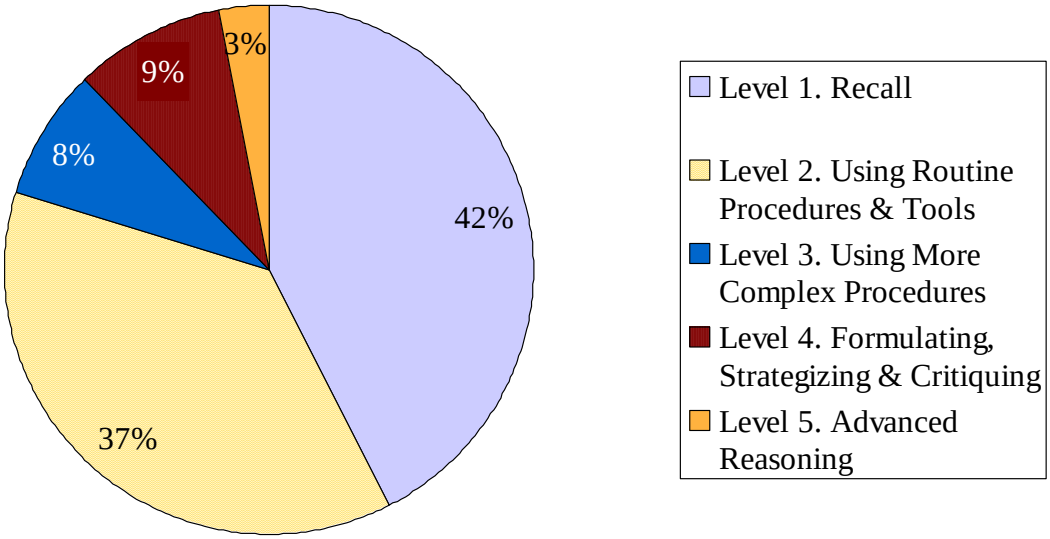
CHART 2: Performance Expectations in Mathematics across the Economies, Grades 1-6



Lower Secondary School: Grades 7-9[xi]

Nearly 80 percent of the performance skills emphasized in grades 7-9 across all economies are from Levels 1 and 2 (Recall and Using Routine Procedures). Just over 20 percent of the performances described in the standards address more cognitively demanding performances, with just 12 percent addressing Levels 4 and 5.

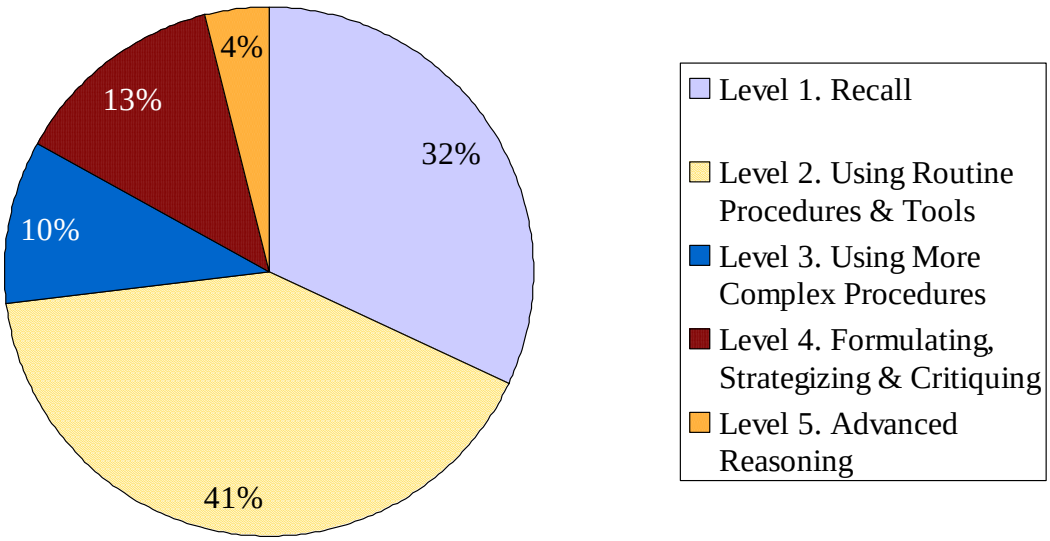
CHART 3: Performance Expectations in Mathematics across the Economies, Grades 7-9



Upper Secondary School: Grades 10-12[xii]

Seventy-three percent of the performances at grades 10-12 consist of Level 1 and 2 expectations, Recall and Using Routine Procedures. The remaining 27 percent of the performances address more cognitively demanding skills, with 17 percent of the emphasis at Levels 4 and 5.

CHART 4: Performance Expectations in Mathematics across the Economies, Grades 10-12



Summary of Performance or Cognitive Skills

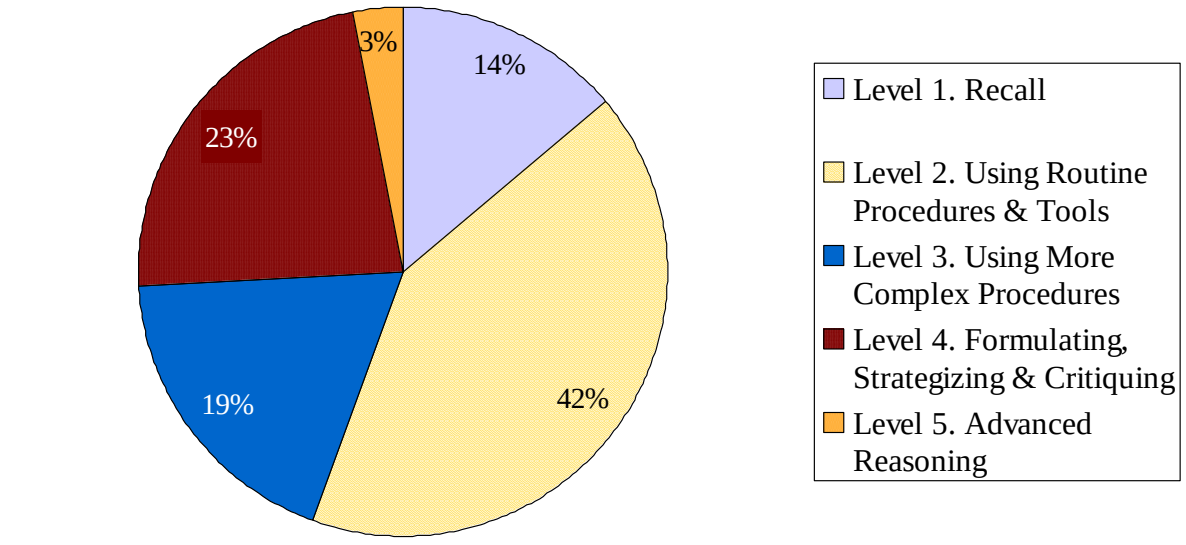
Economies generally emphasize more demanding performance skills at the secondary level. Some trends are more readily apparent when performance skill categories are combined. For instance, as the grade spans progress, basic skills (Recall & Using Routine Procedures) decrease slightly from 82 to 79 to 72 percent. In contrast, the group of skills beyond the rote and routine (Using More Complex Procedures, Formulating Problems and Applying Advanced Reasoning) increases from 19 to 20 to 27 percent. This trend suggests that the level of performance or cognitive skill challenge increases over the grade levels as students work with more advanced content.

Variation among Economies

While on average the economies studied tend to emphasize the Level 1 and 2 skills over the higher-level skills, there is great variation among individual economies. What follows are examples that show the variation in the distribution of performance skill expectations across economies for grade span 10-12. These examples are limited to the expectations set at the individual standard statement level – economies may reinforce, expand or raise expectations to a higher level in other ways (e.g., through curricular guides, instructional materials and assessments).

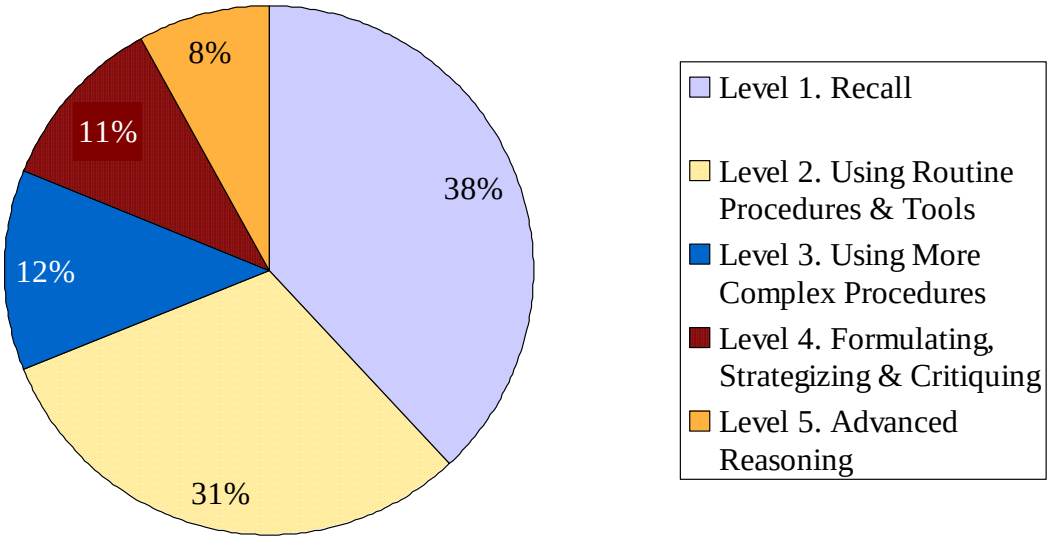
New Zealand dedicates 45 percent of its performance expectations for grades 10-12 to the top three levels, as compared to 27 percent average of all the economies. In the standards from this economy, students are required to critically evaluate data presented by others, make inferences based on data, justify attributes and measures selected, and critique causal relationship claims. Furthermore, they are expected to devise effective solution strategies and to generalize and deduce properties – all of which are higher order cognitive skills.

CHART 5: New Zealand, Performance Expectations in Mathematics, Grades 10-12



China places slightly more emphasis on the top three performance skill categories overall than average of all the economies, but places the heaviest emphasis among the economies – eight percent – on Applying Advanced Reasoning, the highest performance skill. This economy's standards contain a relatively strong emphasis on proof, including reasoning and argumentation. The course component that highlights trigonometry covers derivation of formulae, also a higher order skill. Finally, the standards provide rich opportunities for students to think deeply and logically about algorithms, algorithmic thinking and the connection to computer technology.

CHART 6: China, Performance Expectations in Mathematics, Grades 10-12



Korea's standards use language focused on memorization, basic representation and the performance of routine procedures, with almost no attention given to Using More Complex Procedures or Formulating, Strategizing and Critiquing, with little attention to Applying Advanced Reasoning.

CHART 7: Korea, Performance Expectations in Mathematics, Grades 10-12