Informing Grades 1–6 Mathematics Standards Development: What Can Be Learned From High-Performing with Korea, Singapore, and Hong Kong, China?

Alan Ginsburg, U.S. Department of Education
Steven Leinwand, American Institutes for Research
Katie Decker, University of Iowa

The United States is embarking on a historic policy reversal as it moves toward developing common education standards in reading and mathematics. Supporting this movement is the U.S. Department of Education's $4.3 billion Race-to-the-Top (RttT) competition under the American Recovery and Reinvestment Act (ARRA) and the Common Core State Standards Initiative sponsored by the National Governors Association and the Council of Chief State School Officers. To inform these efforts, this report examines what it means to internationally benchmark mathematics standards for grades 1–6 against the composite standards of three high-mathematics performing Asian economies: Hong Kong, China; Korea; and Singapore.

The U.S. common standards movement offers a unique opportunity to address a well-documented weakness found in many State mathematics standards: many topics are taught in a single grade and many topics are repeated over several grades. This topic spread has led to the well-known characterization of U.S. elementary mathematics curriculum expectations as "a mile wide and an inch deep" (Schmidt, Houang, & Cogan, 2002).

The move to common internationally benchmarked standards offers an opportunity to model U.S. standards off of those of high performing economies such as Singapore, which offer a more coherent and focused set of expectations. The composite Hong Kong, China; Korea; and Singapore standards developed in this report present one effort to internationally benchmark grades 1–6 mathematics standards against high-performing nations.

Methodology

The Hong Kong, China; Korean; and Singapore standards were chosen for international benchmarking because of their high performance on the Trends in International Mathematics and Science Study (TIMSS) assessments and the availability of these standards in English on the Asian Pacific Economic Cooperation (APEC) web site. Because of the concern over lack of rich mathematical content progression with many U.S. state standards, our particular focus in developing a composite set of standards is on learning progressions – how systematically the mathematical content progresses across the grades within a broad mathematical topic. The mathematics standards for the three economies are available at http://hrd.apec.org, the website of the Human Resource Development (HRD) working group of the Asian Pacific Economic Cooperation (APEC), an organization composed of 21 economies bordering the Pacific Ocean.

The development of the composite Korea, Singapore and Hong Kong, China mathematics standards was conducted in three steps: (1) identify the *core mathematics topics* taught within each strand across economies; (2) identify each economy's grade-by-grade *sequence of*

mathematical competencies for each core topic; and (3) create *composite standards* for each core topic by drawing from the learning progressions in the standards of each of the three economies.

Identify the core mathematics topics for each strand. These topics are generally apparent from the topic structure of the standards. The Korean standards present a summary table of "Content Organization" that identifies the major topics. The Singapore and Hong Kong, China standards explicitly identify the topics and subtopics associated with each strand by grade. Table 2 shows the core mathematics topics identified within each strand based on the three sets of standards.

Table 2. Core Mathematics Topics: Grades 1–6

Numbers	Measurement	Geometry	Data/Probability	Patterns/Algebra
• Whole numbers • Addition/ subtraction • Multiplication/ division • Fractions • Decimals • Ratios • Percents	• Linear measurement • Perimeter / area • Volume • Nongeometric measurement – *Time (clock)* – *Time (calendar)* – *Money* – *Weight*	• 2-D shapes • 3-D shapes • Lines • Angles	• Classification of objects by attributes into groups • Pictograms • Bar graphs • Tables • Line graphs • Averages • Pie charts	• Symbols • Equations

For each core topic, identify the learning progressions across grades for that topic from each standard. This underlying concept of learning progressions has been described by the National Assessment of Educational Progress (NAEP), as follows:

> A learning progression is a sequence of successively more complex ways of reasoning about a set of ideas. … In other words, the progression from novice learner to competent learner to expert begins with the acquisition of relevant experiences, principles, concepts, facts, and skills and moves to the accumulation and organization of knowledge in a specific domain and finally to expertise *after extensive experience and practice.* (National Assessment Governing Board, 2008)

The learning progressions for each economy's standards by core topic were constructed by pulling out the content for that core topic grade by grade. For example, the content of the linear measurement topic was identified for each grade by economy.

Create the composite standards. First, the grades covered for each topic were compared across the three sets of standards to ensure similarity. For most topics, the three sets of standards cover the topics over similar grades, although exceptions often occurred at the end points. An illustration of a typical grade pattern is the "measurement of length" presented in Table 3. All three sets of national standards cover length measures in grades 1–3 and Singapore covers measurement of length in grade 5 as a learning objective connected with showing an application of decimals. An outlier grade (e.g., grade 5 Singapore for measurement of length) was included in the composite standard if it was judged to add important content to the learning progression.

Table 3. Measurement of Length by Grade: Hong Kong (H), Korea (K), and Singapore (S)

	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
Length	H, K, S	H, K, S	H, K, S		S	

The composite standards for each core mathematics topic were then created by consolidating the competencies from the three sets of standards that describe what students should know and be able to do across the grades. The composite standards represent a judgment designed to present the essential learning competencies from the three sets of standards that most clearly indicate the learning progression for each topic over the grades. The following rules were employed in creating the composite standards:

- If core topic contents were similar for the three standards grade by grade, the content of the standard that was judged to offer the clearest learning progression was chosen.
- If core topic contents were similar overall across standards but differed in some respects grade by grade, the composite standard reflected a judgment as to which standard offered the most in-depth or clearest learning progression.
- If a core topic contents differed on some competency that was not in the other standards, that competency was included only if it added to and was consistent with the learning progression.

Table 4 shows a sample learning progression for the topic of length within the measurement strand, with some key features:

- The learning progression contains two broad sequences or subtopics, one for concept of length and a second for tools/measuring length.
- The concept of length sequence introduces the concept in grade 1 along with the centimeter unit and proceeds in later grades to present different sizes of measuring units and decimals.
- The tools/measuring length sequence presents similar skills at each of grades 1, 2, and 3, but with a clear progression in the selection of the measurement units used.

Note that the composite standards represent the progression of core learning objectives for a topic and may omit some of the details associated with the standards for a particular topic from any of the three economies.

Table 4. Composite Learning Progression for Length Within Measurement

Measurement Topic	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
Concept of length	• Understand concepts of length and distance • Understand long, longer, longest, short, shorter, shortest • Understand centimeter	• Understand that a meter is greater than a centimeter	• Understand that a kilometer is greater than a meter and that a millimeter is smaller than a centimeter • Understand that—Convert compound units to a smaller or a larger unit		• Convert from a smaller unit to a larger unit and vice versa in decimal form	
Tools/measuring length	• Measure and compare lengths of objects and distance with centimeters • Estimate lengths and distances • Measure length with appropriate tools	• Measure and compare length and distance in meters and centimeters • Estimate lengths and distances • Measure length with appropriate tools	• Measure and compare length and distance in kilometers and millimeters • Estimate lengths and distances • Measure length with appropriate tools			

Sample Learning Progressions

Whole numbers. Counting is one of the first skills that children learn in mathematics. Along with rote, sequential counting, children learn to count and compare the number of objects in sets. Counting progresses with larger numbers over the grades as place value concepts are learned. With larger numbers, children apply their place value understanding to comparing, ordering, and rounding numbers. Numbers are further differentiated into odd and even groups.

Table 5 shows the composite set of standards for whole numbers.

- Grade 1 addresses whole numbers up to 100. Basic whole-number skills include counting the numbers of objects in a set which requires one-to-one correspondence between the objects and the number; comparing the size of sets; ordering numbers; and knowing that numbers show position (1st, 2nd). Place value is introduced to distinguish tens and ones, and the correspondence between numeral symbols and words is taught.
- The grade 2 learning progression extends recognizing and ordering whole numbers up to 1,000 and understanding place value of hundreds, tens, and ones and adding 10 and 100 to numbers mentally.
- Grade 3 focuses on numbers up to 10,000 with place value to thousands.
  At grade 4, numbers are up to 100,000, and rounding and approximation are introduced.
  Grade 5 explicitly treats understanding of large numbers up to a hundred million along with the concepts of approximation, estimation, and rounding.

Table 5. Composite Standards: Numbers—Whole Numbers

Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
Whole numbers to 100: • Count to tell the number of objects in a given set • Count forward and backward • Compare the number of objects in two or more sets • Use ordinal numbers (first, second, up to tenth) and symbols (1st, 2nd, 3rd, etc.) • Use number notation and place values (tens, ones) • Read and write numbers in numerals and in words • Compare and order numbers	Whole numbers to 1,000: • Count in tens and hundreds • Use number notation and place values (hundreds, tens, ones) • Read and write numbers in numerals and in words • Compare and order numbers	Whole numbers to 10,000: • Use number notation and place values (thousands, hundreds, tens, ones) • Read and write numbers in numerals and in words • Compare and order numbers • Understand odd and even numbers	Whole numbers to 100,000: • Use number notation and place values (ten thousands, thousands, hundreds, tens, ones) • Read and write numbers in numerals and in words • Compare and order numbers • Round numbers to the nearest 10 or 100	• Develop an understanding of large numbers • Develop the concept of approximation • Estimate the number of a large quantity of objects • Round large numbers in thousands, ten thousands, hundred thousands, millions, ten millions, hundred millions	

Addition and subtraction. The number standards present addition along with subtraction so that students understand the meanings and relationship. The standards require mastering adding and subtracting smaller numbers, thereby promoting automaticity. The learning progression builds up addition and subtraction skills by introducing successively larger numbers over grades. Word problems are also introduced early on as a way of promoting students' understanding of addition and subtraction concepts (Har & Hoe, 2007). Table 6 shows the composite set of standards for addition and subtraction.

- The concepts of addition and subtraction are introduced in grade 1. Addition and subtraction is initially constrained within 20 and includes learning all the different combinations of sums through 9 plus 9 and finding an unknown number within the combination. By the end of the year, addition and subtraction expand to sums and differences within 100 without regrouping and the introduction of 1-step word problems. Addition and subtraction facts are taught in all three economies using what Singapore calls "number bonds" and the United States calls "fact families" that relate, for example, 4 + 7, 7 + 4, 11 – 4, and 11 – 7 to promote understanding and reduce memory load.
- In grade 2, addition and subtraction are extended to numbers involving three digits, 2-step word problems, and mental calculation.

- In grade 3, addition and subtraction are extended to numbers up to four digits, and again there is a stress on word problems.

Table 6. Composite Standards: Numbers—Addition and Subtraction

Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
Addition and subtraction: • Understand situations for , and the meaning of, addition and subtraction • Use the addition symbol (+) or the subtraction symbol (−) • Compare two numbers within 20 to tell how much one number is greater (or smaller) than the other • Recognize the relationship between addition and subtraction • Build the addition bonds up to 9 + 9 • Solve 1-step word problems involving addition and subtraction within 20 • Add more than two 1-digit numbers • Add and subtract within 100 without regrouping involving – *a 2-digit number and ones* – *a 2-digit number and tens* – *two 2-digit numbers* • Use mental calculation for addition and subtraction – *within 20* – *involving a 2-digit number and ones without renaming* – *Involving a 2-digit number and tens*	Addition and subtraction of numbers up to three digits: • Solve up to 2-step word problems involving addition and subtraction • Use mental calculation for addition and subtraction involving – *a 3-digit number and ones* – *a 3-digit number and tens* – *a 3-digit number and hundreds*	Addition and subtraction of numbers up to four digits: • Use the terms “sum” and “difference” • Solve up to 2-step word problems involving addition and subtraction			

Perimeter and area. Recognizing that perimeter is a measure of length, perimeter and area are treated together because they both measure different attributes of two-dimensional shapes. The three Asian economies differ slightly in when students first encounter these concepts, with Singapore beginning in grade 3 and Korea and Hong Kong, China in grade 4. Our measurement composite elects to follow the Singapore approach beginning in grade 3 because this is consistent with the spread of the learning progression across grades. Table 7 shows the composite set of standards for perimeter and area.

- The concept of perimeter is introduced in grade 3 for regular and irregular two-dimensional shapes. The initial focus of perimeter calculations is on squares and rectangles in tandem with area. Grade 4 adds complexity to the understanding of perimeter by stressing the relationship between perimeter and area wherein students must find one dimension of a rectangle or a square given information about the area or other dimensions of the perimeter. The concept of circumference (the perimeter of a circle) is addressed in grade 6 in combination with the concept of pi and the calculation of the area of a circle.
- The development of the concept of area begins in grade 3 with the introduction of the idea of square units of measurement (square centimeters). By grade 4, the formulas for the areas of rectangles and squares are developed and understanding is deepened by standards that require students to find areas of composite figures made up of rectangles and squares. In grade 5, students are introduced to the area of nonrectangular figures with straight sides, including parallelograms, triangles, and rhombuses. Circles, including finding the area of circles, are introduced in grade 6 after students have been exposed to decimals, which is required for multiplication involving pi.

Table 7. Composite Standards: Measurement—Perimeter and Area

	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr.6
Perimeter				• Develop the concept of perimeter • Measure and find the perimeter of 2-dimensional shapes	• Find one dimension of a rectangle given the other dimension and area and perimeter	• Understand the area and the circumference of circles • Understand the concept of pi • Find the area and the circumference of circles, semicircles, and quarter circles
Area			• Develop the concept of area • Compare areas, using improvised units • Measure area in square centimeters (cm^2) and square meters (m^2)	• Apply the formula for area of squares and rectangles and composite figures made up of rectangles and squares	• Apply the formula for the area of triangles, parallelograms, and rhombuses	

Two-dimensional shapes. The foundation for developing geometric competencies with two-dimensional shapes begins with a recognition of basic two-dimensional figures. The learning progression for two-dimensional figures then expands with the introduction of properties of angles, parallel and perpendicular lines, and symmetry. Table 8 shows the composite set of standards for two-dimensional shapes.

- The grade 1 standards focus on identifying and naming the four basic two-dimensional shapes: rectangle, square, circle, and triangle. Identification includes finding two-dimensional objects in three-dimensional shapes and completing patterns that vary according to the attributes of shape, size, and color.
- The grade 2 standards extend two-dimensional concepts to circles and semicircles. They also include having students physically copy figures on a dot grid and extend geometric patterns by identifying the orientation of shapes in addition to size and color.
- Building on students' exposure to angles and lines in grade 3, the grade 4 standards focus on properties of right angles, including rectangles and squares. Also introduced in grade 4 is the idea of symmetry of two-dimensional figures, including horizontal and vertical symmetry.
- The progression in grade 5 expands to an understanding of the application of angles in different-shaped triangles and incorporating knowledge of right angles and the sum of angles of a triangle. Also addressed is the application of angles and parallel lines to properties of parallelograms, rhombuses, and trapezoids, including sums of angles and construction.

Table 8. Composite Standards: Geometry—Two-Dimensional Shapes

Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
Four basic shapes: rectangle, square ,circle, triangle • Identify and name the four basic shapes from 2-dimensional and 3-dimensional objects, describing and classifying shapes Patterns: • Make or complete patterns with 2-dimensional cut-outs according to one or two of the following attributes: – *shape* – *size* – *color*	• Identify the basic shapes that make up a given figure • Form different 2-dimensional figures with cut-outs of – *rectangle* – *square* – *triangle* – *semicircle* – *quarter circle* • Copy figures on dot grid or square grid Patterns: • Make or complete patterns with 2-dimensional cut-outs according to one or two of the following attributes: – *shape* – *size* – *orientation* – *color*		Rectangle and square: • Understand the properties of a rectangle and a square —Find unknown angles Symmetry: • Identify symmetric figures • Determine whether a straight line is a line of symmetry of a symmetric figure and complete a symmetric figure with respect to a given horizontal or vertical line of symmetry	Triangle: • Identify and name the following types of triangles: – *isosceles triangle* – *equilateral triangle* – *right-angled triangle* • Use the property that the angle sum of a triangle is 180^{o} tp find unknown angles • Draw a triangle from given dimensions using ruler, protractor, and set squares Parallelogram, rhombus, and trapezoid: • Identify and name parallelogram, rhombus, and trapezoid • Understand the properties of parallelogram, rhombus, and trapezoid • Find unknown angles • Draw a square, rectangle,	

				parallelogram, rhombus, or trapezoid from given dimensions using ruler, protractor, and set squares	

Findings

The composite standards have a number of features that can inform an international benchmarking process for the development of K–6 mathematics standards in the U.S.

First, the composite standards *concentrate the early learning of mathematics on the numbers, measurement, and geometry strands* with less emphasis on data analysis and little exposure to algebra. The Hong Kong, China standards for grades 1–3 devote approximately half the targeted time to numbers and almost all the time remaining to geometry and measurement.

Second, the composite standards *sequence topics* within strands to support in-depth and efficient development of mathematics content following a logical development of mathematical knowledge. For example, the numbers strand sequence progression is whole numbers, arithmetic operations, fractions, decimals, ratios, and percents. Measurement introduces linear measurement followed by perimeter and area (two-dimensional measurement) and then more complicated volume (three-dimensional measurement). Geometry initially introduces the features of shapes, proceeds to cover two-dimensional geometry along with angles and parallel lines, and concludes with the features of three-dimensional figures. Data analysis starts with pictograms, a visual and more familiar way to examine data, and then moves on to bar charts and more-complicated continuous line charts.

Third, the composite standards *sequence mathematical competencies within a topic* across the grades according to a mathematically logical progression. Several illustrations occur within the numbers strand. Whole numbers are ordered by size, with grade 1 addressing numbers up to 100, grade 2 up to 1,000, grade 3 up to 10,000, and grade 4 up to 100,000. Grade 5 emphasizes an understanding of large numbers in general. Multiplication is also carefully developed, with grade 2 starting with the basic multiplication concept and multiplication tables for 2, 3, 4, 5, and 10; grade 3 extends to tables 6, 7, 8, and 9 along with multiplication of one digit by two and three digits; grade 4 introduces associative and commutative properties and multiplication of two-digit numbers by three-digit numbers; and grade 5 covers common multiples and the relation with common divisors.

Fourth, the *ordering of content for one topic is frequently aligned to reinforce the content of another topic* for the same or prior grades. Linear measurement in grade 1 introduces the centimeter, which is aligned with grade 1 whole numbers exposure of numbers up to 100. Grade 3 introduces kilometers and millimeters after 1,000 is taught within the whole numbers strand of grade 2. Grade 3 introduces the multiplication and division of money (e.g., relations between total costs with price and quantity), thus reinforcing the learning of multiplication and division in grades 2 and 3. Still another example of cross-topic reinforcement occurs within grade 6 data

analysis, which introduces pie charts around the same time that circles are introduced in geometry.

In addition, it is important to note that, in many cases, particularly within the number strand, the composite standards show a grade placement of a particular skill or concept that is one year earlier than is common in much of the United States. While this is a notable finding, we believe that it is the coherent learning progressions and content connections that are much more important to emulate than the grade placement of particular topics. Furthermore, the delineation of content by learning progressions facilitates an adjustment of the grade placement of content to fit the learning pace of individual students within a common standards framework that all students are eventually expected to master.

In conclusion, standards may only be the front end of a long-term the reform process, but it is critical that sound standards be developed to guide the rest of the reform process. The composite mathematics standards of the three Asian high performers offer a theoretically and empirically valid international benchmark for the development of common U.S. standards in mathematics.

NOTE: The complete version of this paper is available at:
http://www.air.org/news/documents/MathStandards.pdf

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Appendix

Table A1. Composite Standards: Numbers and Operations

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
Whole Numbers / Place Value	Whole numbers to 100: • Count to tell the number of objects in a given set • Count forward and backward • Compare the number of objects in two or more sets • Use ordinal numbers (first, second, up to tenth) and symbols (1st, 2nd, 3rd, etc.) • Use number notation and place values (tens, ones) • Read and write numbers in numerals and in words • Compare and order numbers	Whole numbers to 1,000: • Count in tens and hundreds • Use number notation and place values (hundreds, tens, ones) • Read and write numbers in numerals and in words • Compare and order numbers	Whole numbers to 10,000: • Use number notation and place values (thousands, hundreds, tens, ones) • Read and write numbers in numerals and in words • Compare and order numbers • Understand odd and even numbers	Whole numbers to 100,000: • Use number notation and place values (ten thousands, thousands, hundreds, tens, ones) • Read and write numbers in numerals and in words • Compare and order numbers • Round numbers to the nearest 10 or 100	• Develop an understanding of large numbers • Develop the concept of approximation • Estimate the number of a large quantity of objects • Round large numbers in thousands, ten thousands, hundred thousands, millions, ten millions, hundred millions	
Addition / Subtraction	Addition and subtraction: • Understand situations for , and the meaning of, addition and subtraction	Addition and subtraction of numbers up to three digits: • Solve up to 2-step word problems involving addition	Addition and subtraction of numbers up to four digits: • Use the terms “sum” and “difference”			

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
	• Use the addition symbol (+) or the subtraction symbol (−) • Compare two numbers within 20 to tell how much one number is greater (or smaller) than the other • Recognize the relationship between addition and subtraction • Build the addition bonds up to 9 + 9 • Solve 1-step word problems involving addition and subtraction within 20 • Add more than two 1-digit numbers • Add and subtract within 100 without regrouping involving – *a 2-digit number and ones* – *a 2-digit number and tens* – *two 2-digit numbers* • Use mental	and subtraction • Use mental calculation for addition and subtraction involving – *a 3-digit number and ones* – *a 3-digit number and tens* – *a 3-digit number and hundreds*	• Solve up to 2-step word problems involving addition and subtraction			

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
	calculation for addition and subtraction – *within 20* – *involving a 2-digit number and ones without renaming* Involving a 2-digit number and tens					
Multiplication/Division		Basic multiplication (basic concept and computation): • Understand the situations for, and meaning of, multiplication • Build up the multiplication tables of 2, 3, 4, 5, and 10 • Discover the commutative property of multiplication through concrete examples (e.g., 2 × 3＝3 × 2) Basic division (basic concept and computation): • Develop the concept of division: sharing and grouping	Multiplication: • Build up the multiplication tables of 6, 7, 8, and 9 • Perform multiplication with a multiplier of 1 digit and a multiplicand of 2 or 3 digits Division: • Understand the situations for, and meaning of, division • Perform basic division by short division • Perform division with a divisor of 1 digit and a dividend of 2 or 3 digits with and without remainders	Multiplication: • Discover the associative property of multiplication through concrete examples • Apply the commutative and associative properties of multiplication in computation (e.g., 2 × 8 × 5 = (2 × 5) × 8) • Perform multiplication with a multiplier of 2 digits and a multiplicand of 2 digits and then 3 digits Division: • Perform division with a divisor of 2 digits and a	Divisors and multiples: • Understand the meaning of “divisor,” “common divisor,” and “greatest common divisor” and know how to solve for them • Understand the meaning of “multiple,” “common multiple,” and “least common multiple” and know how to solve for them • Understand the relation between divisors and multiples and know how to apply them • Multiply and divide by 10, 100 and	

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
		• Divide a quantity (not greater than 20) into equal sets given – *the number of objects in each set* – *the number of sets* • Recognize the relationship between multiplication and division • Solve 1-step word problems involving multiplication and division within the multiplication tables	• Use the terms "product," "quotient," and "remainder" • Solve up to 2-step word problems involving the four operations, including estimating answers.	dividend of 2 and then 3 digits • Recognize divisibility when the divisors are 2, 5, and 10 • Identify 1-digit factors of 2-digit numbers • Distinguish between factors and multiples • Solve up to 3-step word problems involving the four operations, including estimating answers	1000 mentally • Use order of operations, combined operations involving the four operations, and brackets • Solve word problems involving the four operations, including estimating answers	
Fractions / Concepts		Fraction of a whole: • Interpret a fraction as part of a whole • Read and write fractions • Compare and order unit fractions and like fractions. (denominators less than or equal to 12)	Equivalent fractions: • Recognize and name equivalent fractions • Write the equivalent fraction of a fraction, given the denominator or the numerator • Express a fraction in its simplest form • Compare and order unlike fractions, including comparing fractions with respect to one half	Mixed numbers and improper fractions: • Understand the concepts of mixed numbers and improper fractions • Express an improper fraction as a mixed number, and vice versa, and expressing both in simplest form		

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
			(denominators less than or equal to 12)			
Fractions/ Arithmetic Operations			Addition and subtraction of two related fractions (one denominator a factor of the other) within one whole (denominators of given fractions should not exceed 12) Multiplication of a proper or improper fraction and a whole number	Addition and subtraction of • like fractions • related fractions (denominators of given fractions should not exceed 12) Multiplication of a proper or improper fraction and a whole number	Addition and subtraction of fractions with unlike denominators: • Add and subtract fractions with unlike denominators Multiplication of fractions: • proper fractions, improper fractions, mixed numbers and whole numbers by proper fractions, improper fractions and mixed numbers • Divide fractions by whole numbers and whole numbers by fractions	Division of fractions: • Divide proper fractions by proper fractions Mixed calculations with fraction and decimal: • Know how to solve simple calculations with both fractions and decimals
Decimals				Decimals up to three decimal places: • Understand notation and place values (tenths, hundredths, thousandths), including identifying the values of the digits in a decimal	Decimal addition and subtraction: • Add and subtract decimals up to two places of decimals and for sums involving at most three operations —Estimate the answers	Decimal division: • Develop an understanding of division of decimals through daily life examples • Divide decimals by whole numbers, whole numbers by decimals, and

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
				• Use the number line to display decimals • Compare and order decimals • Convert a decimal to a fraction • Convert a fraction whose denominator is a factor of 10 or 100 to a decimal • Round decimals to the nearest whole number	Decimal multiplication: • Develop an understanding of multiplication of decimals through daily life examples • Multiply decimals by whole numbers and by decimals • Estimate the answers	decimals by decimals • Perform mixed operations on decimals for sums involving at most three operations • Estimate the answers Decimal conversion: • Convert decimals into fractions and fractions to decimals • Compare fractions by converting them into decimals • Estimate the answers
Ratio					Ratio (excludes ratios involving fractions and decimals): • Interpret *a* : *b* and *a* : *b* : *c*, where *a*, *b*, and *c* are whole numbers • Express a ratio in its simplest form • Find the ratio of two or three given quantities • Write equivalent ratios and find the missing term in a pair of equivalent ratios	

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
					• Solve up to 2-step word problems involving ratio, including finding one quantity given the other quantity and their ratio	
Percents						Percents (basic concept; conversion of percentages into decimals or fractions and vice versa): • Recognize percentages through daily life examples • Develop an understanding of percentages • Convert percentages into decimals and vice versa Applications of percents: • Solve simple problems on percentages, including finding percentages expressing the value of a percentage of a quantity applying discounts • Estimate the

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
						answers
NCTM	Grade 1. Number and Operations: Developing an understanding of whole number relationships including grouping in tens and ones. Children compare and order whole numbers (at least to 100) to develop an understanding of and solve problems involving the relative sizes of these numbers. They think of whole numbers between 10 and 100 in terms of groups of tens and ones (especially recognizing the numbers 11 to 19 as 1 group of ten and particular numbers of ones). They understand the sequential order of the counting numbers and their relative magnitudes and represent numbers on a number line. Number and Operations and Algebra: Developing understandings of addition and subtraction and strategies for basic addition facts and related subtraction facts. Children develop strategies for adding and subtracting whole numbers on the basis of their earlier work with small numbers. They use a variety of models, including discrete objects, length-based models (e.g., lengths of connecting cubes), and number lines, to model "part-whole," "adding to," "taking away from," and "comparing" situations to develop an understanding of the meanings of addition and subtraction and strategies to solve such arithmetic problems. Children understand the connections between counting and the operations of addition and subtraction (e.g., adding two is the same as "counting on" two). They use properties of addition (commutativity and associativity) to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems involving basic facts. By comparing a variety of solution strategies, children relate addition and subtraction as inverse operations. Grade 2. Number and Operations: Developing an understanding of the base-ten numeration system and place-value concepts. Children develop an understanding of the base-ten numeration system and place-value concepts (at least to 1000). Their understanding of base-ten numeration includes ideas of counting in units and multiples of hundreds, tens, and ones, as well as a grasp of number relationships, which they demonstrate in a variety of ways, including comparing and ordering numbers. They understand multidigit numbers in terms of place value, recognizing that place-value notation is a shorthand for the sums of multiples of powers of 10 (e.g., 853 as 8 hundreds + 5 tens + 3 ones). Number and Operations and Algebra: Developing quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction. Children use their understanding of addition to develop quick recall of basic addition facts and related subtraction facts. They solve arithmetic problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition (commutativity and associativity).Children develop, discuss, and use efficient, accurate, and generalizable methods to add and subtract multidigit whole numbers. They select and apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including standard algorithms, for adding and subtracting whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems. Grade 3. Number and Operations and Algebra: Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts. Students understand the meanings of multiplication and division of whole numbers through the use of representations (e.g., equal-sized groups, arrays, area models, and equal "jumps" on number lines for multiplication, and successive subtraction, partitioning, and sharing for division). They use properties of addition and multiplication (e.g., commutativity, associativity, and the distributive property) to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts. By comparing a variety of solution strategies, students relate multiplication and division as inverse					

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
	operations. Number and Operations: Connecting ratio and rate to multiplication and division Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., "If 5 items cost $3.75 and all items are the same price, then I can find the cost of 12 items by first dividing $3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12"). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates. Grade 4. Number and Operations and Algebra: Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication. Students use understandings of multiplication to develop quick recall of the basic multiplication facts and related division facts. They apply their understanding of models for multiplication (i.e., equal-sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (in particular, the distributive property) as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multidigit whole numbers. They select appropriate methods and apply them accurately to estimate products or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including the standard algorithm, for multiplying whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems. Number and Operations: Developing an understanding of decimals, including the connections between fractions and decimals. Students understand decimal notation as an extension of the base-ten system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on. Students relate their understanding of fractions to reading and writing decimals that are greater than or less than 1, identifying equivalent decimals, comparing and ordering decimals, and estimating decimal or fractional amounts in problem solving. They connect equivalent fractions and decimals by comparing models to symbols and locating equivalent symbols on the number line. Grade 5. Number and Operations and Algebra: Developing an understanding of and fluency with division of whole numbers. Students apply their understanding of models for division, place value, properties, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multidigit dividends. They select appropriate methods and apply them accurately to estimate quotients or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including the standard algorithm, for dividing whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems. They consider the context in which a problem is situated to select the most useful form of the quotient for the solution, and they interpret it appropriately. Number and Operations: Developing an understanding of and fluency with addition and subtraction of fractions and decimals. Students apply their understandings of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They apply their understandings of decimal models, place value, and properties to add and subtract decimals. They develop fluency with standard procedures for adding and subtracting fractions and decimals. They make reasonable					

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
	estimates of fraction and decimal sums and differences. Students add and subtract fractions and decimals to solve problems, including problems involving measurement. Grade 6. Number and Operations: Developing an understanding of and fluency with multiplication and division of fractions and decimals. Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement. Number and Operations: Connecting ratio and rate to multiplication and division. Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., "If 5 items cost $3.75 and all items are the same price, then I can find the cost of 12 items by first dividing $3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12"). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates.					

Table A2. Composite Standards: Measurement

	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr.6
Linear measurement:						
Concept of length	• Understand concepts of length and distance • Understand long, longer, longest, short, shorter, shortest • Understand centimeter	• Understand that a meter is greater than a centimeter	• Understand that a kilometer is greater than a meter and that a millimeter is smaller than a centimeter • Understand that—Convert compound units to a smaller or a larger unit		• Convert from a smaller unit to a larger unit and vice versa in decimal form	
Tools/ measuring length	• Measure and compare lengths of objects and distance with centimeters • Estimate lengths and distances • Measure length with appropriate tools	• Measure and compare length and distance in meters and centimeters • Estimate lengths and distances • Measure length with appropriate tools	• Measure and compare length and distance in kilometers and millimeters • Estimate lengths and distances • Measure length with appropriate tools			
Perimeter / Area						
Perimeter				• Develop the concept of perimeter • Measure and find the perimeter of 2-dimensional shapes	• Find one dimension of a rectangle given the other dimension and area and perimeter	• Understand the area and the circumference of circles • Understand the concept of pi • Find the area and the circumference of circles, semicircles, and quarter circles
Area			• Develop the concept of area • Compare areas, using improvised units • Measure area in	• Apply the formula for area of squares and rectangles and composite figures made up of rectangles and	• Apply the formula for the area of triangles, parallelograms, and rhombuses	

	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr.6
			square centimeters (cm^2) and square meters (m^2)	squares		
Volume						
Concept			• Develop the concept of capacity and volume • Understand the need for standardized units of measurement • Understand the units of liter and milliliter		• Introduce the standard unit cubic centimeter (cm^3) • Understand the need for using a unit larger than a cubic centimeter: cubic meter (m^3)	
Tools / Measuring			• Measure and compare the volumes of containers using liter and milliliter • Measure volume with appropriate tools		• Measure and compare objects using cm^3 • Understand and apply the formula for finding the volume of cubes and cubes	• Find the length of one edge of a cube given its volume or find the height of a cube given its volume and base area
Nongeometric Measurement						
Time: Clock	• Tell and write time to the hour and half hour	• Tell and write time to 5 minutes • Use a.m. and p.m.	• Tell and write time to 1 minute • Solve word problems involving adding and subtracting time down to the minute	• Measure time in seconds • Use a 24-hour clock to solve clock word problems		
Time: Calendar	• Learn the days of the week • Recognize that there are 12 months in a year	• Recognize the number of days in a month and a year • Understand the relation among 1 hour, 1 day, 1				

	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr.6
		week, 1 month, and 1 year				
Money	• Tell the amount of money in cents up to $1 and in dollars up to $100 (Excludes combination of dollars and cents)	• Read and write money in decimal notation	• Solve problems involving adding and subtracting money in decimal notation	• Solve word problems involving the four operations and money in decimal notation		
Weight	• Develop the concept of weight • Compare the weights of concrete objects • Express heavy and light and use the terms "heavy," "heavier," and "heaviest"	• Understand the need for using standard units • Measure and compare the weights of objects using gram (g) and kilogram (kg) • Choose the appropriate tools for measuring			• Convert a measurement from a smaller unit to a larger unit in decimal form and vice versa, using kilograms and grams	
NCTM	*Measurement* and *Data Analysis: (Grade 1)* Children strengthen their sense of number by solving problems involving measurements and data. Measuring by laying multiple copies of a unit end to end and then counting the units by using groups of tens and ones supports children's understanding of number lines and number relationships. Representing measurements and discrete data in picture and bar graphs involves counting and comparisons that provide another meaningful connection to number relationships. *Measurement:* Developing an understanding of area and determining the areas of two dimensional Shapes (Grade 3) Students recognize area as an attribute of two-dimensional regions. They learn that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps. They understand that a square that is 1 unit on a side is the standard unit for measuring area. They select appropriate units, strategies (e.g., decomposing shapes), and tools for solving problems that involve estimating or measuring area. Students connect area measure to the area model that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle. *Geometry* and *Measurement* and *Algebra:* Describing three-dimensional shapes and analyzing their properties, including volume and surface area (Grade 5) Students relate two-dimensional shapes to three-dimensional shapes and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces. Students recognize volume as an attribute of three-dimensional space. They understand that they					

	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr.6
	can quantify volume by finding the total number of same-sized units of volume that they need to fill the space without gaps or overlaps. They understand that a cube that is 1 unit on an edge is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume. They decompose three-dimensional shapes and find surface areas and volumes of prisms. As they work with surface area, they find and justify relationships among the formulas for the areas of different polygons. They measure necessary attributes of shapes to use area formulas to solve problems.					

Table A3. Composite Standards: Geometry

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
2D- Shapes	Four basic shapes: rectangle, square ,circle, triangle • Identify and name the four basic shapes from 2-dimensional and 3-dimensional objects, describing and classifying shapes Patterns: • Make or complete patterns with 2-dimensional cut-outs according to one or two of the following attributes: – *shape* – *size* – *color*	• Identify the basic shapes that make up a given figure • Form different 2-dimensional figures with cut-outs of – *rectangle* – *square* – *triangle* – *semicircle* – *quarter circle* —Copy figures on dot grid or square grid Patterns: • Make or complete patterns with 2-dimensional cut-outs according to one or two of the following attributes: – *shape* – *size* – *orientation* – *color*		Rectangle and square: • Understand the properties of a rectangle and a square —Find unknown angles Symmetry: • Identify symmetric figures • Determine whether a straight line is a line of symmetry of a symmetric figure and complete a symmetric figure with respect to a given horizontal or vertical line of symmetry	Triangle: • Identify and name the following types of triangles: – *isosceles triangle* – *equilateral triangle* – *right-angled triangle* • Use the property that the angle sum of a triangle is 180° tp find unknown angles • Draw a triangle from given dimensions using ruler, protractor, and set squares Parallelogram, rhombus, and trapezoid: • Identify and name parallelogram, rhombus, and trapezoid • Understand the properties of parallelogram, rhombus, and trapezoid • Find unknown angles • Draw a square, rectangle,	

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
					parallelogram, rhombus, or trapezoid from given dimensions using ruler, protractor, and set squares	
3-D Shapes	• Recognize prisms, pyramids, and spheres. • Identity 3-dimensional shapes intuitively • Group 3-dimensional shapes • Describe the relative positions of two 3-dimensional shapes briefly • Make or complete patterns with 3-dimensional models, including, cube (rectangular block), cone, and cylinder	• Identify prisms, cylinders, pyramids and cones • Recognize faces • —Group 3-dimensional shapes • Make 3-dimensional shapes • Form different 3-dimensional figures with concrete models of – *cube* – *cone* – *cylinder*				• Understand the concepts of prisms and pyramids and their components and properties • Work with various solid figures: – *Looking at a solid figure made by building blocks, count the number of blocks used* – *Make various shapes using building blocks, and find the patterns* – *Express the shape of a solid figure made by building blocks from the top, front, and side* • Identify nets of the following solids: cube, prism, and pyramid and make

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
						3-dimensional solids from given nets • Understand the concepts of cylinders and cones and their components and properties • Understand the concept of a solid of revolution
Lines		• Identify lines (straight lines) and curves • Identify edges and faces of a 3-dimensional object	• Identify and draw perpendicular and parallel lines			
Angles			• Identify angle as an amount of turning • Identify angles in 2-dimensional and 3-dimensional objects • Identify right angles, angles greater than/smaller than a right angle	• Use notation such as ∠ABC and ∠x to name angles • Estimate and measure angles in degrees • Draw an angle using a protractor • Associate 1/4 turn/right angle with 90°; 1/2 turn with 180°; 3/4 turn with 270°; complete turn with 360°; and 8-point compass		
NCTM	Grade 1. Geometry: Composing and decomposing geometric shapes Children compose and decompose plane and solid figures (e.g., by putting two congruent isosceles triangles together to make a rhombus), thus building an understanding of part-whole relationships as well as the properties of					

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
	the original and composite shapes. As they combine figures, they recognize them from different perspectives and orientations, describe their geometric attributes and properties, and determine how they are alike and different, in the process developing a background for measurement and initial understandings of such properties as congruence and symmetry. Grade 3. Geometry: Describing and analyzing properties of two-dimensional shapes Students describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to definitions of shapes. Students investigate, describe, and reason about decomposing, combining, and transforming polygons to make other polygons. Through building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space and the use of those attributes and properties in solving problems, including applications involving congruence and symmetry. Grade 5. Geometry and Measurement and Algebra: Describing three-dimensional shapes and analyzing their properties, including volume and surface area Students relate two-dimensional shapes to three-dimensional shapes and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces. Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that they need to fill the space without gaps or overlaps. They understand that a cube that is 1 unit on an edge is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume. They decompose three-dimensional shapes and find surface areas and volumes of prisms. As they work with surface area, they find and justify relationships among the formulas for the areas of different polygons. They measure necessary attributes of shapes to use area formulas to solve problems.					

Table A4. Composite Standards: Data Analysis

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
Classifying Objects	• Classify objects or people by a predetermined standard, and count the numbers in each category.					
Pictograms		• Compare the quantity of three or more types of objects by arranging them in lines • Read, construct and interpret picture graphs with scales • Solve problems using information presented in picture graphs				
Bar Graphs			• Read/discuss block graphs in which 1 square represents 1 unit, average value • Read, construct and interpret bar graphs in both horizontal and vertical forms, including using their scales • Solve problems using information presented in bar graphs			

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
Tables				• Complete a table from given data • Read and interpret tables • Solve problems using information presented in tables		
Line Graphs				• Collect data of continuous variates and express them in a graph of broken lines • Compare bar graphs and the graphs of broken lines to understand the properties and uses of each graph		
Averages					• Interpret average as "total amount ÷ number of items" • Calculate the average number/quantity • Solve word problems involving average, including finding the total amount given the average and the number of items	

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
Pie Charts						• Read and interpret pie charts • Solve 1-step problems using information presented in pie charts
NCTM Focal Points	Gr. 1. Measurement and Data Analysis: Children strengthen their sense of number by solving problems involving data. Representing measurements and discrete data in picture and bar graphs involves counting and comparisons that provide another meaningful connection to number relationships. Gr. 3. Addition, subtraction, multiplication, and division of whole numbers come into play as students construct and analyze frequency tables, bar graphs, picture graphs, and line plots and use them to solve problems. Gr. 4. Students continue to use tools from grade 3, solving problems by making frequency tables, bar graphs, picture graphs, and line plots. They apply their understanding of place value to develop and use stem-and-leaf plots. Gr. 5. Students apply their understanding of whole numbers, fractions, and decimals as they construct and analyze double-bar and line graphs and use ordered pairs on coordinate grids.					

Table A5. Composite Standards: Algebra

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
Expressions					Expressions: • Use symbols or letters to represent numbers • Record with algebraic symbols, for example, "John is *x* years old now. How old will he be after 10 years?" and record as (*x* + 10) years old	
Equations					Simple equations involving 1 step in finding the solution: • Understand the concept of equations • Solve simple equations involving 1 step in the solutions and check the answers (involving whole numbers only) • Solve problems by simple equations (involving only 1 step in the solutions)	Simple equations (involving 2 steps in finding the solution): • Solve equations involving at most 2 steps in the solutions, and examine the results • Solve problems by using simple equations (involving at most 2 steps in the solution)
NCTM Focal Points	Grade 6. Writing, interpreting, and using mathematical expressions and equations Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information). Students know that the solutions of an equation are the values of the variables that make the equation true. They solve simple					

Topics	Gr. 1	Gr.2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
	one-step equations by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation. They construct and analyze tables (e.g., to show quantities that are in equivalent ratios), and they use equations to describe simple relationships (such as 3x = y) shown in a table.					