Formative Assessment: a key component in improving student achievement in mathematics in New Zealand

Ian Stevens, Ministry of Education

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Introduction

> "To be numerate is to have the ability and inclination to use mathematics effectively – at home, at work and in the community.[1]"

The goal of mathematics education in New Zealand is for all students to become numerate, leaving school with a positive attitude towards mathematics, coupled with an understanding and ability to use mathematics effectively whenever needed.

The New Zealand national curriculum[2] is the official policy relating to teaching and learning in schools. It is a statement of what New Zealand deems important in education. Its principal function is to set the direction for student learning and to provide guidance for schools as they design and review their curriculum. It takes as its starting point a vision of young people as lifelong learners who are confident and creative, connected, and actively involved. It sets out values that are to be encouraged, modeled, and explored, defines five key competencies that are critical to sustained learning and effective participation in society and describes the outcomes for students in eight interconnecting learning areas.

Mathematics is a key learning area or subject in the New Zealand curriculum and essential to students becoming confident and creative, connected and active learners. The learning area in New Zealand is called mathematics and statistics. Mathematics is the exploration and use of patterns and relationships in quantities, space, and time and statistics is the exploration and use of patterns and relationships in data. These two disciplines are related but are different ways of thinking about and solving problems. Both equip students with effective means for investigating, interpreting, explaining, and making sense of the world in which they live.

The Third International Mathematics and Science Study (TIMSS) in 1994/1995 identified the achievement of New Zealand students as being significantly below the international mean in mathematics and science[3]. In response to these results the government established the Mathematics and Science Taskforce to provide advice on how to improve the teaching of mathematics and science in New Zealand schools. The taskforce highlighted a number of overriding priorities in relation to raising performance in mathematics, in particular the need to raise teachers and parents expectations of success, improve the professional skills, knowledge and confidence of teachers, provide resources and professional development for teachers to

[1] The definition and goal of numeracy, New Zealand Ministry of Education, 2002

[2] *The New Zealand Curriculum*, 2007 for English medium schools and *Te Marautanga o Aotearoa*, 2008 for Māori medium schools, Ministry of Education. The terms English medium and Māori medium are used to indicate the language of instruction.

[3] New Zealand's TIMSS results can be found at http://www.educationcounts.govt.nz/publications/series/2571

support them in implementing the curriculum, and lift Māori and Pacific Island students' levels of achievement. These strategic priorities led to the design and implementation of the Numeracy Development Project[4]. The project was developed for the two languages of instruction used in New Zealand schooling English medium, starting in 2000, and Te Poutama Tau, Māori medium, starting in 2002. Each of the projects informs the ongoing development of the other.

A system wide focus on improvement was adopted, rather than a focus on specific groups of students or regions. This was in response to the diversity of New Zealand students and schools. "New Zealand has a wide spread of achievement compared to other highly performing economies – with relatively large proportions at both a very high level and also at a very low level."[5] Students from all ethnicities, socio economic backgrounds and genders are represented in both the highest and lowest performing groups in New Zealand. However, Māori students, Pacific Island students and students from low socio economic backgrounds are proportionally over represented in the lowest performing group. The number of small schools in New Zealand, many rural and isolated, is another feature that needed to be considered in planning for improvement.[6]

Numeracy Development Project

The Numeracy Development Project and Te Poutama Tau, major government funded national strategies, aim to improve student achievement through improving the professional capability of teachers.

The strategic objectives of the Numeracy Development Project and Te Poutama Tau are:

- Improved student achievement in mathematics,
- Improved knowledge, skills and confidence of teachers in mathematics,
- Improved achievement of Māori and Pacific Island students, and
- Māori language revitalization (Te Poutama Tau).

A dynamic and evolutionary approach to the design and implementation of the project is a key feature, with assessment, research and evaluation used to inform the ongoing development at the classroom, school and system level. The design drew on evidence from mathematics education, effective teaching, teacher learning, effective professional development, educational change and system reform as well as from the on-going research and evaluation associated with the project. Since the beginning of the project there have been 103 research and evaluation papers published by the Ministry of Education[7].

The premise of the project to improve student achievement by improving the professional capability of teachers is based on the belief that teachers are key figures in changing the way in which mathematics is taught and learned in schools. Their subject matter, pedagogical knowledge and assessment capability are critical factors in the teaching and learning of mathematics. The effective teacher has a thorough and deep understanding of the subject matter

[4] The terms numeracy and mathematics are used interchangeably in New Zealand with specific information located in the Mathematics and Statistics learning area of *The New Zealand Curriculum*.
[5] Satherley, (2010)
[6] Almost 50% of New Zealand's primary schools have less than 150 students and almost 20% of schools have only two teachers. 5 to 13 years old students attend New Zealand primary schools.
[7] Papers can be downloaded from www.nzmaths.co.nz/annual-evaluation-reports-and-compendium-papers

to be taught, how students are likely to learn it, the difficulties and misunderstandings they are likely to encounter, and effective formative assessment practices.

The location of the professional development was also a key feature in the design of the project. "Professional learning is strongly shaped by the context in which the teacher practises. This is usually the classroom, which, in turn, is strongly influenced by the wider school culture and the community and society in which the school is situated. Teachers' daily experiences in their practice context shape their understandings, and their understandings shape their experiences."[8] School advisers, external to the school, support teachers and school leaders by leading workshops, visiting teacher's classrooms to model ideas with students, observing and giving feedback. They also providing resources, assist in the analysis of student achievement information, and support the learning needs of the school's teachers and leaders as needed.

The professional development model for the Numeracy Development Project and Te Poutama Tau is based around teachers understanding and using three key pedagogical tools;

- The Number Framework,
- Diagnostic Interview, and
- Strategy Teaching Model.

These three tools together enable teachers to developing their knowledge, ability and confidence in knowing their students learning needs, and be able to provide a quality teaching and learning programme. The number framework provides a structure for teaching and learning, the diagnostic interview finds out where on the framework students are, as well as the next learning steps, and the strategy teaching model guides how to teach this next step.

The project starts with teachers being introduced to the number framework through workshops, which includes videos of students articulating their thinking. Teachers conduct the diagnostic interview with each student in their class, initially with support from the adviser. The resulting student achievement information is used to develop a teaching programme based on the learning needs of the students. Through a series of workshops and classroom visits by the adviser, teachers gradually improve their professional capability. Their teaching becomes based on the learning needs of their students rather than on a predetermined programme based on the age of the students or level of schooling.

The Number Framework

The structure of the framework is based on the idea that there are increasingly sophisticated ways of thinking mathematically and that it is useful to set out the different types of thinking as a progression for pedagogical purposes.

The framework is divided into two main sections, strategy and knowledge, each with eight stages of development. The strategy section describes the mental processes students use to solve problems involving numbers and estimate answers. The knowledge section describes the key items of knowledge that students need to learn and be able to quickly recall in order to be able to estimate and solve problems. It is important that students make progress in both sections of the framework. Strong knowledge is essential for students to broaden their strategies across a full

[8] Timperley, 2008

range of numbers, and knowledge is often an essential prerequisite for the development of strategies. The strategy section is based on two broad areas of development, the first based on counting and the second on the notion of part-whole thinking, (Cobb & Wheatly, 1998).

The framework is an important pedagogical tool for teachers as it enables them to become more focused in their teaching through developing their knowledge of how students learn mathematics. Often at the start of the professional development, teachers indicate their vagueness about what they were teaching in mathematics. Contrasting their previous practice, they commented, "I am much more focused in my teaching objectives", "The project has given my teaching more structure." and "It's about giving simple, understandable, credible, reasonable structures for teachers to use." [9] Laying out professional progressions in some detail in the framework enabled in-depth assessment of students' understanding of mathematical ideas which teachers found very helpful, (Higgins & Parsons, 2009).

The Diagnostic Interview

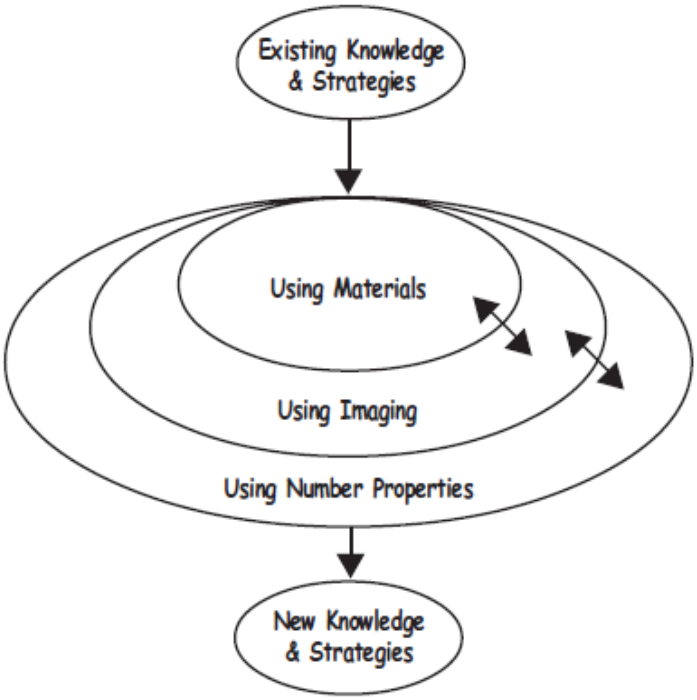
One of the outcomes sought through teachers' participation in the project is increased teacher responsiveness to students' diverse learning needs through seamlessly integrating knowledge of number progressions into their mathematics teaching practice. The diagnostic interview, based on Wright's (1998) work, has been designed to support teachers' development in identifying students' knowledge and strategies and using the evidence as the basis for planning students' next learning sequence. The information gained can also be used to report to parents. The fact that items in the diagnostic interview are aligned to the number framework provides teachers with an enriched knowledge about progressions in learning number. It is "one of the essential triggers" for challenging teacher's beliefs and changing teacher's knowledge and practice, (Higgins & Parsons, 2009).

One of the most powerful outcomes for teachers when first using the diagnostic interview was the overturning of previously held assumptions about the extent of individual student understanding of number concepts. Many teachers commented that through the interview they found that some students whom they had previously thought to have good number understanding struggled in their attempts to explain their answers. Conversely, other students, whom teachers had regarded as having weak knowledge, demonstrated deeper understanding. A teacher expressed how conducting the diagnostic interview with her own students affected her preconceived ideas, (Higgins & Parsons, 2009). Teachers started to think about what students had learnt rather than what they had been taught. Comments like "This group of students won't be able to achieve like the others", "They won't know this because I haven't taught it yet" and "They will know this because I have just taught them" were challenged. Teachers started to shift from focusing on how many answers students got correct to how students worked them out and their level of understanding. The framework and interview gave teachers a clear picture of each student's understanding and ability. This led to the need to develop classroom programmes that were responsive to the students' actual learning needs.

The Strategy Teaching Model

[9] Higgins and Parsons, 2009

The strategy teaching model is interconnected to the framework and the information gained from the diagnostic interview guided teachers in the explicit teaching of strategies (Hughes, 2002). The following diagram outlines the model. The development of any new strategy starts from what the student already knows and can do. The arrows on the diagram illustrate a dynamic relationship between the phases with movement through these phases demonstrating greater degrees of abstraction in a student's thinking. Progression from Using Materials to Using Imaging is usually promoted by the teacher masking materials and asking anticipatory questions about actions on those materials. Progression to Using Number Properties is promoted by increasing the complexity or size of the numbers involved, thus making reliance on the material representation difficult and inefficient. This model was influenced by the P-K theory of Pirie & Kieren (1989).



A feature of the model is the aim of developing students' mental strategies through students explaining their thinking prior to the introduction of the written algorithm. Shifting teachers' practice from the premature introduction of the written algorithm has implications for the organization of classroom learning, the use of presentations, the recording of problem solutions, and parent expectations which need to be considered and planned for.[10]

The framework, interview and model are key parts of a quality teaching and learning programme along with the seven dimensions of quality teaching critical to improving student outcomes identified by research. The dimensions are incorporated throughout the project resources,

[10] Numeracy Development Project, Book 3, Ministry of Education, 2008

discussed at workshops and school visits, and modelled by advisers in classrooms. The seven dimensions outlined by Alton-Lee, 2003[11] are:

- Inclusive classroom climate
- Focused planning
- Problem-centred activities
- Responsive lessons
- Connections
- High expectations
- Equity

The project is supported by a wide range of resources, including the nine Numeracy Development Project books[12] and the Ministry of Education's mathematics curriculum website, www.nzmaths.co.nz. The website contains a wide variety of documents, videos, online professional development opportunities, planning tools, interactive learning tools, lessons and units of work. There is also a small section to support parents in helping their children. The website has both English medium and Māori medium sections.

Implementation Approach

Since 2000, almost all of New Zealand's 2,100 primary schools have been involved in the initial two years of the Numeracy Development Project. This is approximately 29,000 teachers, including approximately 800 Māori medium[13] teachers, and 800,000 students. The average time allocated to each teacher for facilitation and support by an adviser is approximately 13 hours in the first year and 5 hours in the second year, costing approximately $3,300 per teacher. This cost is used to contract and coordinate teams of school advisers to work with teachers and school leaders, to release teachers to conduct the diagnostic interview, to provide resources and equipment, and fund research and evaluations.

The project is centrally coordinated from the national office of the Ministry of Education and regionally led by coordinators with teams of advisers all working together. The regional teams are based at the six main New Zealand universities allowing for synergy between pre-service teacher educators, researchers and school advisers. Access for teachers to university post-graduate mathematics education papers through a fee subsidy is also provided.

Advisers work directly alongside teachers the first time they conducts the diagnostic interview to guide and support their interpretation of the students' responses to items from the interview. The diagnostic interview has three embedded design elements: First, it is designed as a model for the types of questions that teachers might use in teaching students; second, teachers deepen their understanding through the items in diagnostic interview which illustrate the different stages of the number framework; and third, the information gained through the interview enables teachers to develop more specific expectations of student learning. The strategy and knowledge components of the interview build teachers' knowledge of the interconnectedness of mathematical ideas, (Higgins & Parsons, 2009).

[11] Numeracy Development Project, Book 3, Ministry of Education, 2008
[12] Numeracy Development Project books can be downloaded at: http://www.nzmaths.co.nz/numeracy-development-projects-books
[13] The language of instruction is Māori, the language of the indigenous people of New Zealand

Following the diagnostic interview a cycle of workshops and classroom and school visits begins. The workshops follow the order of the project books. Following each workshop advisers visit each teacher's classroom to model the ideas from the workshop. The in-class work of the adviser varies over the course of the project in response to the teachers learning need. The adviser's in-class work includes modelling lessons or parts of lessons, teaching alongside the classroom teacher, and observing the teacher in action followed by feedback and discussion. Supporting planning at both the classroom and school level is also an important role of the adviser along with support the school's mathematics leaders as needed. Initially the adviser takes a lead in the development, alongside the school leaders who are encouraged to be fully involved. As the implementation progresses the school leaders take over leading the development in their school. Near the end of each year the schools leaders, with advisers support if needed, plan ways to sustain the improvements already made and to continue improving teacher capability and student achievement.

To continue supporting schools after being involved in the initial two years of the project, regionally based networks of numeracy lead teachers from each school are organized by advisers. All schools are invited to send one or more lead teachers to these regular meetings where they hear about new resources, new ideas and how others are improving teacher capability and student achievement. This forum also allows lead teachers to support and learn from each other as well as hear from advisers and other experts.

Evaluating the Effectiveness of the Project

A research and evaluation programme investigating the effectiveness of the Numeracy Development Project has played a critical role in the success of the project. The approach adopted to gather evidence has been multi methodological and iterative with a focus on student achievement, the professional practice of teachers and advisers, and sustainability, (Higgins & Parsons, 2009).

Each year researchers are contracted by the Ministry of Education to research and evaluate the project. The research includes studies analysing changes in student achievement and trends in the data over time. As part of the project teachers enter student achievement information in relation to the framework onto a secure website. This information is used to evaluate the project and in planning future professional development. Other studies focus on teacher and adviser practice and the longitudinal effects of the project. This research has both an English medium and a Māori medium setting focus.

The following table is an example of how one Numeracy Development Project school reported the progress in achievement during the first year to their Board of Trustees. The table shows the number of students at each stage of the Number Framework in each year level at three points in the year; February, June, and November.[14]

The blue in the table indicates the level of achievement expected at the end of that year, yellow indicates that they are just below the expected level, grey indicates that they are above the expected level and green indicates that they are "at risk" or sufficiently below the expected level

[14] In New Zealand most students start school on their fifth birthday and have their eighth birthday while in Year 3 and eleventh birthday while in Year 6.

that their future learning in mathematics is in jeopardy. The school also used similar tables to analyze the progress and achievement of their Māori students and compare boys and girls achievement at the end of the year.

Numeracy Student Achievement Information: All Students at School X

Curriculum Levels	Numeracy Stages	Year 1			Year 2			Year 3			Year 4			Year 5			Year 6		
	Time of year	Feb	Jun	Nov	Feb	Jun	Nov	Feb	Jun	Nov	Feb	Jun	Nov	Feb	Jun	Nov	Feb	Jun	Nov
Level 5	Stage 8																		2
Level 4	Stage 7															2			2
Level 3	Stage 6									4			6	1	4	10	4	9	19
Level 2	Stage 5		1				1	1	5	12	2	5	25	8	25	20	19	20	14
	Stage 4		1	6		4	26	9	13	20	13	31	13	18	12	13	17	13	7
	Stage 3	1	1	9	7	11	11	10	16	5	26	9	1	16	5	4	5	4	2
	Stage 2	6	14	10	17	23	7	15	8	3	4			6	3		0		
Level 1	Stage 1	5	10	1	16	7		9	3	1	0			0			1		
	Emergent	15		1	6	1	1	1											
	Totals	27	27	27	46	46	46	45	45	45	45	45	45	49	49	49	46	46	46
At or above expectation		26%	63%	93%	0%	9%	59%	22%	40%	80%	4%	11%	69%	18%	59%	65%	9%	20%	50%
Cause for Concern		74%	37%	7%	52%	74%	39%	22%	36%	11%	29%	69%	29%	37%	24%	27%	41%	43%	30%
At Risk		0%	0%	0%	48%	17%	2%	56%	24%	9%	67%	20%	2%	45%	16%	8%	50%	37%	20%

Above Expectation | Expectation stage for class | Cause for Concern | At Risk

This evidence based approach has been supported by the development and publication of *Effective Pedagogy in Mathematics/Pāngarau* as part of the Ministry of Education Best Evidence Synthesis Programme[15]. The quality of this synthesis has been recognised through its publication by the International Academy of Education as part of its *Educational Practices Series*[16].

Formative Assessment in Action

The role of formative assessment or assessment for learning[17] is an integral component of the Numeracy Development Project. At the heart of the project's philosophy are teachers listening, watching, noticing and talking with students, with the information gained used to develop or modify classroom programmes based on the learning needs of the students.

For example, a teacher plans a lesson to teach a group of students how to subtract groups of ten from any three-digit number in their heads, e.g. 214 – five tens. At the start of the lesson the teacher gives the students a short activity to check their existing knowledge and notices that the students have trouble with questions like, "9 tens + 24" or "sixty three + 70". Using this

[15] www.educationcounts.govt.nz/themes/BES

[16] www.ibe.unesco.org/en/services/publications/educational-practices.html

[17] "Assessment for Learning is part of everyday practice by students, teachers and peers that seek, reflects upon and responds to information from dialogue, demonstration and observation in ways that enhance ongoing learning" Draft position paper from the Third International Conference on Assessment for Learning, Dunedin, New Zealand, March 2009.

information the teacher quickly modifies the lesson to focus on helping the students to understand and become confident with adding groups of ten to a number. The teacher first asks the students to explain how they worked out their answers, listening, watching and reflecting to work out what to do to help the students. The responses of the students guide the teacher throughout the lesson.

Formative assessment is not only used at the classroom level. The adviser who starts a "How to teach decimals" workshop by gathering feedback from the teachers about the previous workshop on teaching fractions is also gathering information that can be used in a formative way. Finding that the teachers have many questions and are confused about teaching fractions, the adviser modifies the workshop to focus on answering the teachers rather than proceeding with the predetermined plan is an example of formative assessment in action or assessment being used for learning. The adviser's decision to change the workshop is based on their knowledge that the way they were going to teach decimals is based on teachers fully understanding how to teach fractions. The adviser decided on the spot that without a full understanding there was no point in proceeding as planned as it would lead to even more confusion, best to help the teachers understand how to teach fractions before moving ahead.

Modifying the way things are done also occurs at the project level. An example of this in New Zealand was the commonly held belief that understanding place value, (hundreds, tens and ones) would not be a problem for students studying mathematics in Māori medium settings. This belief arises out of the fact that the Māori language itself assists with this understanding, for example, 27 is "rua tekau ma whitu" which literally means two tens and seven. However student achievement research findings showed that this belief was not true. This research finding resulted in a change to the Te Poutama Tau to a deliberate focus on place value. Significant improvements in student's understanding of place value were reported in the following year's research findings.[18]

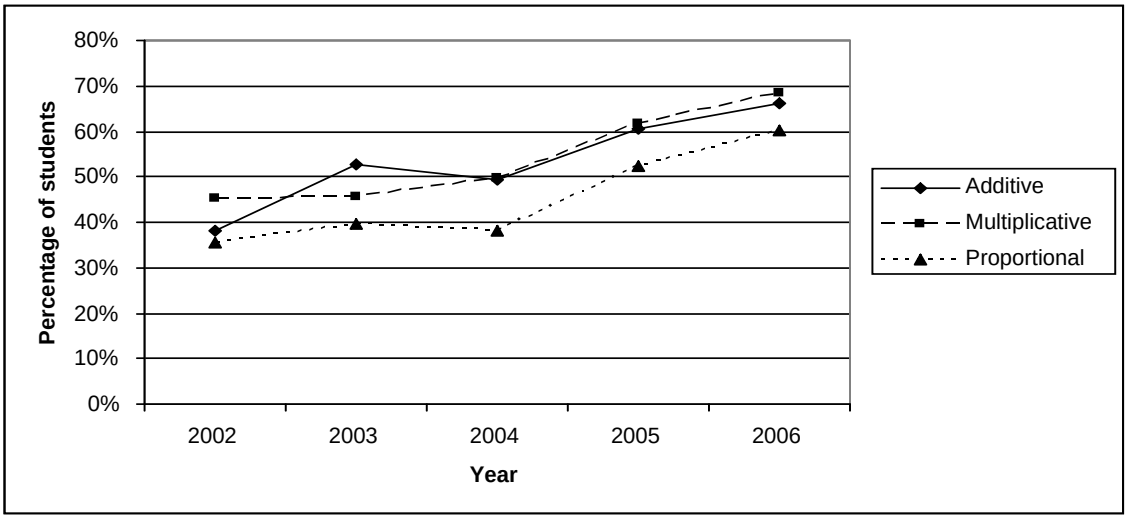
Challenges and opportunities

There is no doubt that the Numeracy Development Project and Te Poutama Tau have been effective in improving student outcomes. Overall student achievement has improved over the years of the project, and the disparities between the achievements of different ethnic groups are reducing.[19]

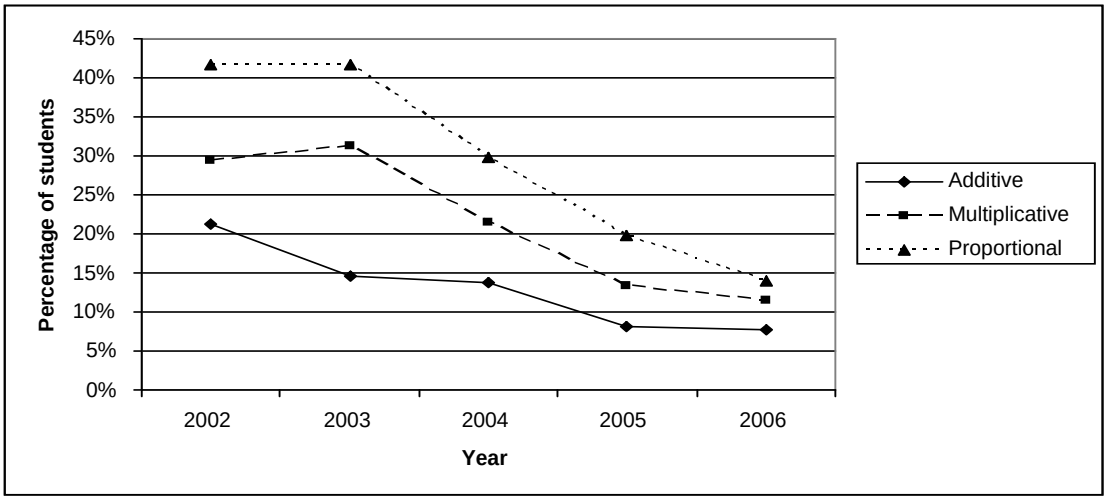
The following graphs show the improvement in the percentage of Year 6 students at or above the expected level, and the reducing percentage well below the expected level[20] at the end of the year. These are students in this study have been in classrooms with teacher who have recorded numeracy achievement information at the end of each year at school from Year 1 through to Year 6, i.e. their stage of achievement from the Number Framework. Most Year 6 students are 11 years old at the end of Year 6.

[18] Christensen (2004)
[19] Satherley, (2010)
[20] Thomas and Tagg, (2008)

Percentage of Year 6 students at or above the expected stage



Percentage of Year 6 students rated as well below the expected stage



In the ten years it has been operating, the project has provided a unique opportunity to develop an understanding about the design of powerful professional development that improves student outcomes. The pedagogical tools of the number framework, the diagnostic interview, and the strategy teaching model are critical elements of the professional development design. The integration of these elements ensures a focus on the core ideas of improving teacher knowledge of mathematics, enhancing understanding of how students learn mathematics, assessment capability and enhancing understanding of how to represent mathematical concepts.

From a system perspective, the outcomes of improving the quality of teaching and student achievement through an unrelenting focus on the core of teaching practice – curriculum, assessment, and pedagogy – in the context of the teacher's own classroom has provided the opportunity to learn about scaling up professional development provision, while maintaining the capacity to effect deep and consequential change. This large-scale case study in mathematics education is evidence that all students benefit when pre-service teacher educators, researchers, school advisers, teachers, and policy makers work together for educational reform, using what is known from research to design and deliver powerful professional development, (Higgins & Parsons, 2009).

Currently New Zealand is introducing National Standards in mathematics. The National Standards in mathematics themselves and the implementation plan have been designed to build on the Numeracy Development Project. A solid platform has been laid over the ten years of the project so that National Standards can be integrated and used to further develop and improve mathematics education in New Zealand. By continuing this approach of aligning and interconnecting all parts New Zealand is moving closer to having all students leave school with a positive attitude towards mathematics, coupled with an understanding and ability to use mathematics effectively whenever needed.

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