

## **Principles and Processes for Publishing Textbooks and Alignment with Standards: A Case in Singapore**

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**Abstract:** Mainly drawing on the author's experience in textbook development for Singapore schools and research in this area, this paper presents six principles and discusses relevant processes for developing mathematics textbooks. These principles include curriculum principle, discipline principle, pedagogy principle, technology principle, context principle, and presentation principle. For each principle, the author briefly explains what it means, why it is important, and how it can be implemented for the development of mathematics textbooks.

**Key Words:** Singapore mathematics; Mathematics curriculum; Textbook development

Over the last 15 or so years, Singapore students' outstanding performance in mathematics in large-scale international comparative studies has generated considerable interest among educational researchers and policy makers in its approaches to school mathematics education. Its modern mathematics textbooks, as a most important resource in support of teaching and learning in mathematics classroom, have also received much attention. In fact, Singapore school mathematics textbooks, considered to some extent as exemplary ones, have been adopted, with or without modification, in quite a number of economies over the last decade (e.g., see Quek, 2002). Having said this, I must add that, as researchers have found, there is still much room in Singapore mathematics textbooks for further improvement (e.g., see Ng, 2002; Fan & Zhu, 2007).

In this paper, I shall propose six inter-related principles and discuss relevant processes for developing (publishing) mathematics textbooks. For brevity, these basic principles are termed curriculum principle, discipline principle, pedagogy principle, technology principle, context principle, and presentation principle, respectively, as shown in Figure 1 below. For each principle, I shall briefly explain what it means and/or signifies, why it is important, and how it can be implemented for the development of mathematics textbooks. The discussions are mainly based on relevant research work I, and my co-researchers, have done (e.g., see Fan & Zhu, 2000, 2002, 2007; Ng, 2002; Lee & Fan, 2004), and personal experiences I gained as consultant/editor-in-chief for both primary and secondary mathematics textbooks over the last ten years. In particular, when it is helpful I will draw on examples in the new series of secondary school textbooks, *New Express Mathematics* (see Fan, Cheng, Dong, Leong, Lim-Teo, Ng, et al., 2007, 2008), to illustrate my discussion.

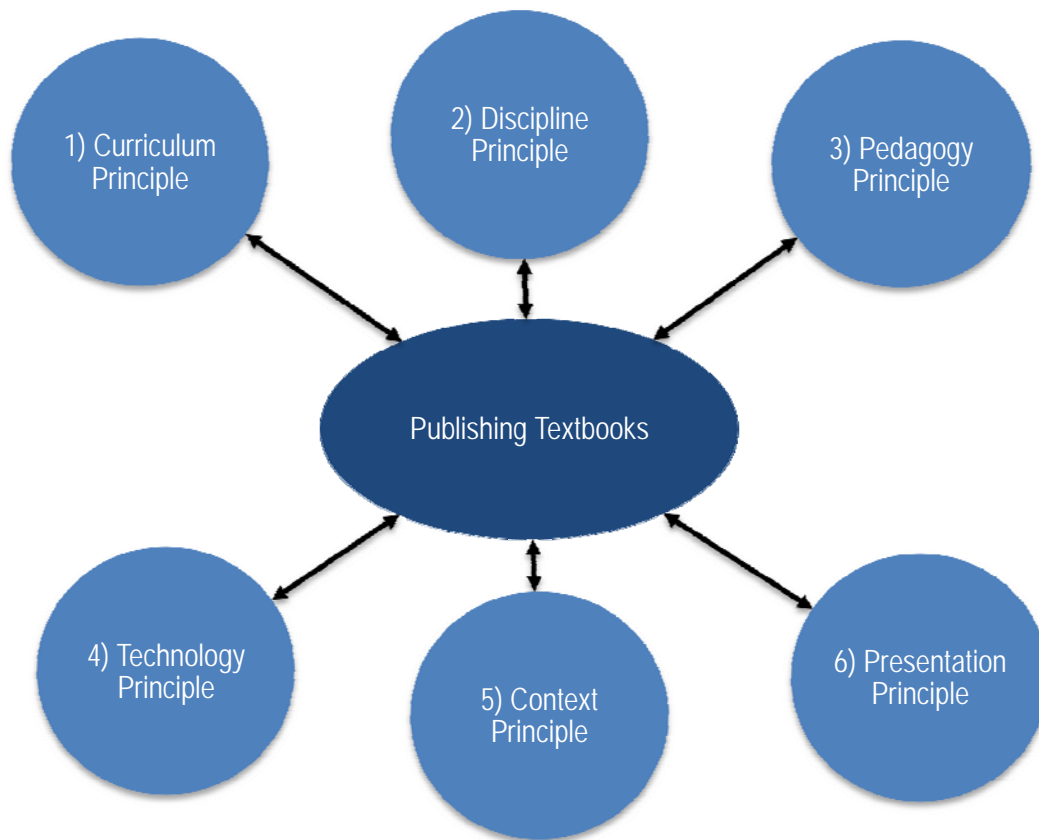


Fig. 1 Six Principles for Developing Mathematics Textbooks

It should be noted that Singapore is an island city-state and has a highly developed economy with a GDP per capita of US\$38,904 in 2008 (Source: Singapore Department of Statistics). It has a population of about 5 million and an area of about 700 square kilometers. Chinese, Malay, Tamil, and English are all official languages, but English is the most widely used working language and the medium of instruction in schools. Singapore adopts a highly centralized education system.

### **Principle 1: Curriculum Principle**

The curriculum principle requires that textbooks must be developed for the implementation and realization of intended curriculum.

In a broad sense, curriculum is a course of study, or all the experiences a student will have and achieve in school. School textbook developers (or authors/writers) must have certain “intended” curriculum, explicitly or inexplicitly, in their minds before they develop their textbooks.

In modern societies, because of a variety of reasons and needs, many (if not all) economies have developed so-called national curriculum (syllabus or standards) for school mathematics. In Singapore, like in many other Asian economies, national mathematics curriculum (syllabus) is developed and issued by the Ministry of Education (MOE), and all schools are required to follow the syllabus in teaching, learning, and assessment. Accordingly, textbooks must align themselves with the syllabus. Below is the well-known Singapore mathematics curriculum framework, also known as pentagon framework, stated in the national syllabus (MOE, 2006a, 2006b).

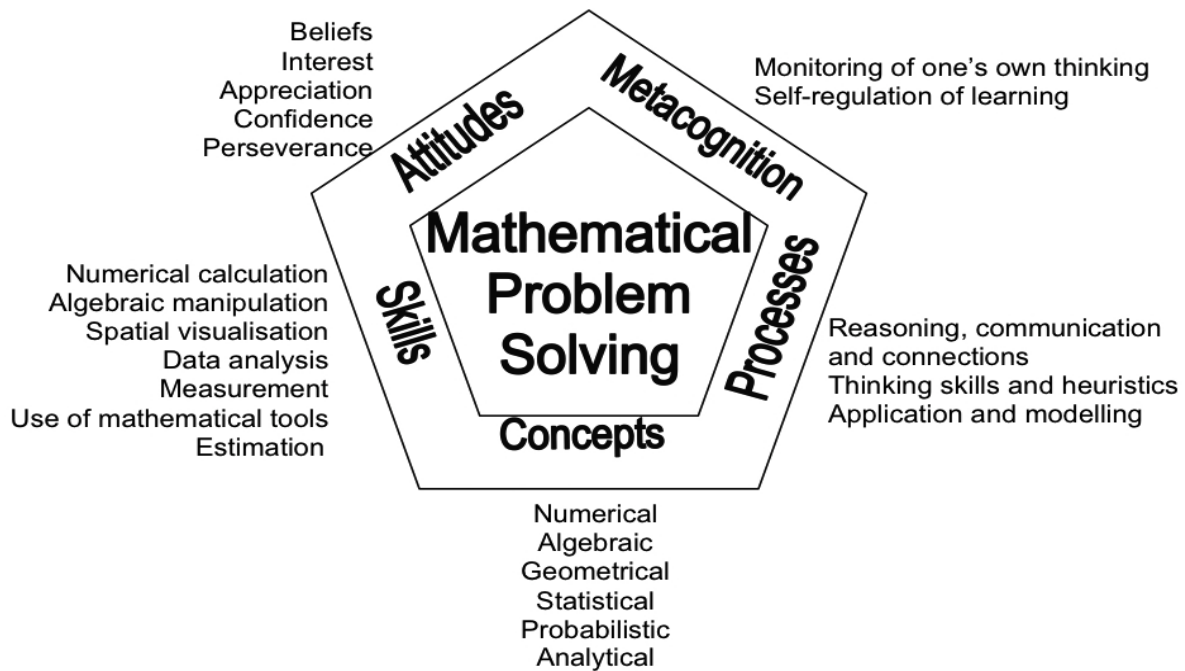


Fig. 2 Singapore Mathematics Curriculum Framework

To align textbooks with the curriculum in terms of the coverage of contents, as roughly reflected in “Concepts” and “Skills” in the pentagon framework and detailed in the syllabus, is important and, relatively, easy. What is more challenging is for textbooks to reflect other aspects that the curriculum intends to achieve, for example, developing students’ high-order thinking skills, critical thinking skills and creativity, and positive attitudes towards mathematics, etc. In fact, Ng (2002) found that the whole series of primary textbooks developed by CDIS (see below) only introduced 11 out of the 14 problem solving heuristics listed in the syllabus. Similar inconsistency was also found in secondary mathematics textbooks for the lower grade level, i.e., Grade 7 and Grade 8 (Fan & Zhu, 2007).

As textbooks are essentially textbook developers’ own interpretation and reflection of the intended curriculum in the process of textbook development, they must study and hence establish good knowledge of the curriculum, and more importantly, work together and get information and feedback from curriculum developers.

With regard to the alignment of textbooks with national curriculum, as pointed out by Kho, who is a most senior curriculum specialist of MOE, during an interview conducted as a preparation for this paper, an exceptional case in Singapore was that in the 1980s and early 1990s, all the primary mathematics textbooks and a set of secondary mathematics textbooks were developed by two specially appointed teams who also developed the syllabus in the Curriculum Development

Institute of Singapore (CDIS) under MOE (Kho, personal communication, Jan. 31, 2010). In other words, the curriculum developers are also textbook developers<sup>1</sup>.

Since the mid 1990s, due to a number of reasons the development and publication of textbooks have been decentralized in Singapore. Nevertheless, the curriculum developers have always maintained very close working connection and interaction with the textbook developers through a variety of activities and channels, including curriculum briefings (e.g., see Mathematics Unit, 2004, 2005), seminars, and meetings. More importantly, all school textbooks in Singapore must be reviewed and approved, primarily based on the curriculum, by a evaluation committee appointed by the Ministry of Education before they can be published and used in schools, and understandably, the curriculum developers have always played a leading role in the evaluation committee. In addition, the textbook developers must revise their textbooks according to the feedback given in the review report. I think these practices have worked very successfully in Singapore and are worth recommendation.

### **Principle 2: Discipline Principle**

There is no doubt that mathematics is a very mature and well-established scientific or academic discipline. The discipline principle requires that school mathematics textbooks must provide solid foundation for the students to understand, apply, and study mathematics in their daily life, further learning and workplace. In terms of content, textbooks must correctly present mathematics knowledge (including mathematical concepts, facts, and methods, etc.). Furthermore, also more challengingly, textbooks should properly represent and reflect the nature, the structure, and epistemology of mathematics as a discipline.

The importance of the discipline principle in developing textbooks is easy to see, but to implement it is not as easy as people might think. Many studies on textbooks have indicated a surprisingly large number of cases in which the textbooks presented the content improperly or incorrectly (e.g., see Levin, 1998).

My own experiences in textbook study and development also suggest that many problems found in textbooks are technical and can be corrected easily, but there are still many which are non-technical or conceptual and they pointed to the problems or weakness in the knowledge base of the textbook developers.

Just to give one case, I shall use an example in the topic of synthetic division in algebra. Many advanced school and college algebra textbooks explicitly stated that this method is only applicable to a divisor in the form of  $x-a$ , and it cannot be extended to a divisor being a polynomial with degree higher than 1 (e.g., see Larson & Hostetler, 1997), which is incorrect.

In connection with this topic, to introduce the long division as shown below, some textbooks place the quotient at the top of  $0x^3+2x^2-7x-10$  instead of  $5x^5+13x^4+0x^3+2x^2$ . Although either way will produce the correct answer at this level, the former will hinder students' further learning about how the method can be generalized for other kinds of divisors with degree more

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<sup>1</sup> The similar practice also existed in China for a long time, where People's Education Press was the organization both to stipulate the national syllabus and to develop/publish the textbooks based on the syllabus. The situation was changed in the late 1990s, when a new round of curriculum reform was launched.

than 1, and hence latter expression should be used (for more details about the synthetic division and its generalization, see Fan, 2003).

$$\begin{array}{r}
 5x^3 + 3x^2 + 9x - 7 \\
 x^2 + 2x - 3 \overline{) 5x^5 + 13x^4 + 0x^3 + 2x^2 - 7x - 10} \\
 \underline{5x^5 + 10x^4 - 15x^3} \\
 3x^4 + 15x^3 + 2x^2 \\
 \underline{3x^4 + 6x^3 - 9x^2} \\
 9x^3 + 11x^2 - 7x \\
 \underline{9x^3 + 18x^2 - 27x} \\
 -7x^2 + 20x - 10 \\
 \underline{-7x^2 - 14x + 21} \\
 34x - 31
 \end{array}$$

From the fact that numerous studies have consistently revealed that many mathematics teachers don't have sufficient knowledge for effective teaching of mathematics (e.g., Carpenter, Fennema, Peterson, & Carey, 1988; Fan, 1998; Ma, 1999), the situation here is not surprising, although it should be noted that virtually no study has been done about what knowledge textbook developers need and have. This is worth attention from researchers as well as policy makers.

With regard to the discipline principle, it is clear that textbook developers must have a sound knowledge base in mathematics as a discipline. It is also very helpful, whenever possible, to have mathematicians in the textbook development team, particularly for secondary and higher level textbooks. In fact, in a latest series of secondary mathematics textbooks (of which I served as chief editor), 10 of the 16 my fellow developers (authors) are trained mathematicians, holding PhD degrees in mathematics from reputable universities. It makes us have more confidence in claiming that one of the key features of the textbooks is, "content is mathematically sound" (Fan, et al., 2007, 2008).

Another relevant point is that textbook developers must carefully collect feedback from the teachers and students after they have used the textbooks. Many times, the problems and mistakes in textbooks cannot be totally detected until they are really used in schools. It implies that textbook development should be ideally an ongoing process.

Having textbooks reviewed, especially by mathematicians and school teachers, is also important in terms of this principle. In Singapore, as Kho pointed out, being mathematically correct is one of the basic criteria for the reviewers to make recommendation for the approval of the textbooks (Kho, personal communication, Jan. 31, 2010), and the textbook developers must correct the mistakes and address the concerns, if any, raised by the reviewers in this aspect.

### Principle 3: Pedagogy Principle

The pedagogy principle requires that textbooks must be developed to facilitate the teaching, learning, and assessment in mathematics.

As Fan and Kaeley (2000) indicated, textbooks as a learning tool or resource can convey different pedagogical messages to teachers (and students) and provide them with an encouraging or discouraging curricular environment, promoting different teaching (and learning) strategies. In fact, available studies have consistently revealed, textbooks can, to different extent, affect not only what to teach, but also how to teach, which will ultimately affect students' learning in mathematics (Zhu & Fan, 2002; Fan, Chen, Zhu, Qiu, & Hu, 2004).

In usual, the pedagogical orientation can be provided in the textbooks implicitly, but sometimes it is helpful to make some pedagogical messages explicit. For example, in *New Express Mathematics*, the authors labeled some sections with headings such as "Let us try", "Looking back", "In-class activity", and "Project task" to make the message about the learning and learning process more explicit. For assessment, the textbooks classified mathematics questions into Group A, B, and C. Journal writing tasks and other kinds of so-called alternative assessment tasks are also provided in the textbooks (Fan, et al., 2007, 2008).

Regarding this principle, as found in the case of Singapore, textbook developers are often given more room to be flexible in pedagogical matters. It is important that textbook developers keep abreast with the new development of the practice, theories, and research in pedagogy and learning. It is also very helpful to have mathematics educators and mathematics teachers in the textbook development team, particularly for developing the textbooks for students at lower grade levels. While mathematics educators have strengths in pedagogical theory and research, school teachers often know better the practices and needs of teachers and students in schools. In *New Express Mathematics* mentioned above, all the other authors are mathematics educators, and most of them have school teaching experiences (Fan, et al., 2007, 2008).

It is worth mentioning that in the process of developing the primary mathematics textbooks by CDIS in the 1980s, as said earlier, all the content and activities designed by the project team, which was led by Kho, were piloted in classrooms in a number of schools which volunteered to participate in the trial, and then revised according to the feedbacks from the try-out before they were finalized and published. According to Kho (personal communication, Jan. 31, 2010), this process was unique and very effective for the developers to make sure that the textbooks being developed would be suited to the needs of teaching and learning in classrooms.

Having textbook reviewed by mathematics educators, or pedagogical experts, and school teachers before the textbooks are published and listening to teachers' feedback after publication is also important for improvement with respect to the pedagogy principle.

#### **Principle 4: Technology Principle**

The meaning of technology in mathematics education has expanded over the time, from calculator, to calculator and computer, and now more commonly to information and communication technology (ICT).

About 15 years ago, I criticized, with good intention, that mathematics education including textbooks in China was largely isolated from modern technology and there was virtually no

existence of technology in the mathematics textbooks (Fan, 1995). I must say that this criticism is no longer valid, as China has made dramatic progress in this aspect in the new wave of curriculum reform, most visibly in the new textbooks developed. In Singapore, much progress has also been made over the last decade or so.

Undoubtedly, the advent of modern technology has produced significant influences on our modern society. In the field of mathematics education, technology has affected what to teach and how to teach, and moreover, why to teach. In relation to this, technology must be reflected and, more importantly, embedded into the teaching and learning of mathematics. Textbooks, as a most important pedagogical resource, must integrate technology to support and facilitate the teaching and learning of mathematics. With the rapid development of technology, it appears apparent that technology will play an increasingly important role in the next generation of mathematics textbooks.

Let me briefly share some examples in the case of *New Express Mathematics* to illustrate how the technology principle is, to different extents, reflected in the textbook development. The first example is, in the older mathematics textbooks of which I was also a consultant/general editor, the approximate value of  $\pi$ , most commonly  $22/7$  and sometimes 3.142, was provided mainly for easy calculation. This is no longer the case in the new textbooks, because all students are expected to use calculators, in which keying in  $22/7$  is not only redundant and less accurate, but also less efficient than directly keying in the symbol “ $\pi$ ” or “pi” (similar idea applies to other special values in mathematics, e.g., e).

Another example is that, as all students in Singapore are expected to have access to ICT including calculators, computer and internet, the textbooks developers have developed more authentic and challenging problems including investigative and project tasks. In working on these problems, students will focus more on conceptual understanding, information gathering, logical reasoning and data analysis, and so on, rather than tedious calculation, complex algebraic manipulation, or time-consuming drawing, etc. By doing so, technology can make mathematics teaching and learning not only more efficient, but also more effective. In fact, many questions in the textbooks that are targeted to develop students’ high-order thinking and problem solving abilities are ICT-embedded.

In addition, many topics covered in the textbooks, especially those in geometry (e.g., for measuring and construction) and statistics (e.g., for statistical diagrams, graphical representation and data analysis), were introduced with the use of available mathematics software to facilitate students’ learning (Fan, et al., 2007, 2008).

The technology principle requires that the textbook developers be familiar with the development of technology. In particular, having experts in the use of ICT in mathematics teaching and learning on board would be most helpful in this aspect. In addition, feedback from teachers and students is also helpful with regard to this principle.

Finally, I shall very briefly mention context principle and presentation principle. Although I think in some sense they are less important and more technical compared to these described earlier, they are still worth reasonable attention in textbook development.

## **Principle 5: Context Principle**

School mathematics textbooks are not research publications for pure mathematics, which can be almost completely abstract. School mathematics is often contextualized, and cannot be free from the social and cultural background, under which school education takes place.

The context principle requires the textbook developers provide adequate cultural, social and even historical contexts when introducing mathematics concepts and contents. This principle is particularly important when application of mathematics is concerned. In the case of *New Express Mathematics*, many examples and problems use Singapore's local context as background. For example, they use authentic information about Singapore's geography (e.g., for distance, speed, and time), demography and economy (e.g., for statistics), architecture (e.g., for geometrical shapes), and society (e.g., social welfare and public housing system for financial mathematics). The contextualized information is provided to motivate and engage students in their learning of mathematics as they are familiar and can make connection with the contexts. In this sense, as it is found in the case of Singapore mathematics textbooks, modification or localization of the textbooks is necessary when they are developed in one economy but used in other economies.

The context principle requires the textbook developers have reasonable knowledge of local contexts. Having local mathematics education experts and school teachers in the development team is important in this aspect. Searching information from local newspapers and other sources can also be very helpful.

### **Principle 6: Presentation Principle**

This principle requires that the presentation of the contents in textbooks must suit the level and needs of teaching and learning. This principle is meaningful in the textbook development as well-designed presentation can make the reading and use of textbooks easy and pleasant, and facilitate teaching and learning.

The principle is more about the technical aspects of developing and publishing a textbook, "design and physical features". In Singapore, the mathematics unit of the Ministry of Education once recommended the following four aspects for textbook developers/publishers to consider: 1. Real-life pictures and realistic drawings, 2. Clear layout and illustrations, 3. Use of colors, and 4. Simple language. (Mathematics Unit, 2004).

Largely consistently, in developing the series of *New Express Mathematics*, the developers' guidelines in this aspect were "1. Use clear and concise language to describe mathematics concepts and process, so it is easy for students to understand; and 2. Use diagrams, pictures, and other visual representations, whenever possible, to make the textbooks more interesting and visually appealing to students and hence enrich and enhance students learning experiences in mathematics" (New Express Mathematics Project Team, 2004).

To implement this principle, textbook developers and publishers should work together (and share the strengths and responsibilities), have experts or specialists in relevant areas, and most importantly, pay reasonable attention to the aspects as highlighted above.

The following table provides a summary of the principles and process/recommendations for developing mathematics textbooks, as presented and discussed in the article. Readers are reminded again that they are based on my own experience and mainly with a Singapore context.



Table 1

*A Summary of Principles and Processes for Publishing Mathematics Textbooks*

Basic Principles	Processes/Recommendations
<b>Curriculum Principle:</b> Textbooks must be developed for the implementation and realization of intended curriculum.	<ol style="list-style-type: none"> <li>1. Textbook developers have good knowledge of the curriculum.</li> <li>2. Textbook developers closely work together and have interaction with curriculum developers.</li> <li>3. Textbooks be reviewed by reviewers including curriculum developers.</li> </ol>
<b>Discipline Principle:</b> Textbooks must provide solid foundation for the students to understand, apply, and study mathematics.	<ol style="list-style-type: none"> <li>1. Textbook developers have a sound knowledge base in mathematics.</li> <li>2. Development team include mathematicians.</li> <li>3. Textbook developers collect ongoing feedback from the users.</li> <li>4. Textbooks be reviewed by reviewers including mathematicians and school teachers.</li> </ol>
<b>Pedagogy Principle:</b> Textbooks must be developed to facilitate the teaching, learning, and assessment in mathematics.	<ol style="list-style-type: none"> <li>1. Textbook developers have good knowledge in pedagogy.</li> <li>2. Developer team include mathematics educators and school teachers.</li> <li>3. Textbook developers collect ongoing feedback from the users.</li> <li>4. Textbooks be reviewed by reviewers including pedagogical experts and school teachers.</li> </ol>
<b>Technology Principle:</b> Textbooks must integrate technology to support and facilitate the teaching and learning of mathematics.	<ol style="list-style-type: none"> <li>1. Textbook developers be familiar with the development of technology.</li> <li>2. Developer team include experts in the use of ICT in mathematics teaching and learning.</li> <li>3. Textbook developers collect feedback from the users.</li> </ol>
<b>Context Principle:</b> Textbooks must provide adequate cultural, social and even historical contexts when introducing mathematics concepts and contents.	<ol style="list-style-type: none"> <li>1. Textbook developers have good knowledge of local contexts.</li> <li>2. Development team include local mathematics experts and school teachers.</li> </ol>
<b>Presentation Principle:</b> Textbooks must suit the level and needs of teaching and learning.	<ol style="list-style-type: none"> <li>1. Textbook developers and publishers work together.</li> <li>2. Development team include experts in relevant areas.</li> </ol>

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