Taking Off With Numeracy: Helping students to catch up

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Taking Off With Numeracy (TOWN) is the current title of a program designed to address the persistence of highly inefficient methods of calculating. Imagine a 12-year-old student being told that there are 56 people coming to a party and that each table at the party can seat 8 people. If the student is then asked how many tables are needed for the party, you might not be surprised that the student reaches for pencil and paper to work it out. However, if the student then starts to make marks on the paper for each person, counts and circles each group of eight before finally counting the number of groups, you have seen an inefficient method of counting by one used to solve a division problem.

Figure 1. Using inefficient methods for division

Count-by-one strategies are a normal part of the development of children's mathematical knowledge. Over time children usually develop a range of methods other than counting by one (Carpenter & Moser, 1982; Fuson, 1988; Steffe, Cobb, & von Glaserfeld, 1988; Wright, 1991; Wright, Martland, Stafford, & Stanger, 2002). For some students the pathway to more advanced methods of working with numbers is passed by and they persist with count-by-one methods (Gray, 1991). In a study of mixed ability children aged 7 to 12, Gray referred to the dominance of strategies of counting by ones in use by less able students, and concluded that… *in one sense they make things more difficult for themselves and as a consequence become less able* (1991, p. 570). Gray's comments emphasise the dangers of persistent counting by ones and why these students are on the wrong path—they are on a road to nowhere!

One of the difficulties with inefficient strategies is that, although they are often much slower, they still work. This means that inefficient strategies persist for some children long past the time that they are useful. Indeed, some children simply get faster at using the inefficient method of counting by one. While these methods will often result in the correct answer, they take so much effort that there is little chance of learning new material. The learning of these students in mathematics has reached a plateau.

Taking Off With Numeracy operates as both a whole class program and a within class intervention. The first phase of implementing the program is the identification phase. This involves determining students' current understandings of multi-unit (conceptual) place value. Students' responses to a short assessment are analysed and categorised in relation to what they reveal about students' thinking using a *Place value framework*. The same assessment is used to determine the target group for the in-class intervention. The students in the intervention group are then given a further individual assessment to determine if their problems also relate to specific reading difficulties.

Two assessment schedules are used in the TOWN whole class screening process, one for students in Years 3 and 4, and the other for students in Years 5 and 6. It takes students approximately 15 minutes to complete the initial assessment. To get the most information out of the assessment, the students should be encouraged to show their thinking for solving each task on the sheet. The analysis of the assessment does not focus on the number of correct answers but rather looks at the methods students are using.

The addition of an in-class intervention is provided for classes with significant numbers of students who make extensive use of count-by-one methods. Once the targeted intervention group has been tentatively identified based on the results of the whole-class assessment, a short follow-up individual assessment is administered. The follow-up assessment may also be used with any students where the teacher feels the initial results are not definitive. That is, the follow-up assessment process may also be used for those students whom the teacher was unable to allocate to or exclude from the target group, or for any student in the class about whom the teacher deems it would be useful to find out further information. During the follow-up assessment, students are asked to explain their strategies for solving problems.

The process also involves the use of *Newman's Error Analysis* to identify the point at which students are experiencing difficulty in solving a *word problem*. Through this process, teachers will be able to determine if the students are experiencing difficulty with reading the question, comprehending the question or with the numeracy involved in the question.

Why focus on place value?

Students often develop a simple structural approach to place value. That is, when a student says the "4" in "48" means "four tens", she or he may be demonstrating only verbal knowledge based on the left and right positional labels associated with the digits. Using this approach, a student may not recognise that the "4" represents 40 objects . This distinction between verbal syntactical knowledge and conceptual (semantic) understanding of place value has implications for developing number sense and teaching algorithms based on place value.

The following example of "trading" shows the dominance of routine over understanding.

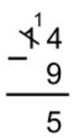


Figure 2. When trading doesn't help

The student might explain this as, "Nine from four you cannot do, so I trade one ten, nine from fourteen is five". That is, the explanation of trading does not help to solve the problem as it results in additional work with no gain as the final question remains ‘nine from fourteen’. This calculational explanation is different from a conceptual explanation and it is possible that the student is only restating what he or she has heard.

Similarly, a student who does not have a well-developed sense of place value might remember the procedure for addition depends on lining up numbers. To answer 27 + 8, the student writes:

$$\begin{array}{r} 27 \\ \underline{+\ 8} \end{array}$$

Figure 3. Creating the appearance of an algorithm

The procedure followed is then to count on the 8, incrementing the seven by ones to reach 15, writing down the 5 and *carrying* the one. The process of incrementing by ones is usually accompanied by making dots or some form of tally system. This procedure of lining numbers up in columns often passes for an understanding of *place value*.

Rather than relying solely on counting by ones, students need to be assisted to use collection-based methods. Using collection-based procedures can make use of:

doubles, 6 + 6 = 12,

number facts, 6 + 3 = 9,

and tens within place value to "bridge to ten", 19 + 7 = (19 + 1) + 6.

The teaching of algorithms and the use of mental strategies for the four operations with numbers, all rely on an understanding of place value. When this understanding is weak, students use partially remembered procedures or else revert to methods of counting by ones.

Place value understanding also has a significant impact on the use of decimals and related work in measurement. For example, the persistence of unitary counting procedures can result in students being restricted to counting squares to find area or counting individual cubes to determine volume. They cannot successfully use the groupings necessary to determine area or volume efficiently.

Multi-unit place value

Multi-unit place value is evident when students can flexibly regroup hundreds, tens and ones. The student’s concept of ten is central to the development of base ten place value. When students first construct number, they construct a system of ones. Therefore, for most students in Kindergarten and Year 1 in Australia, 24 is 24 ones. At the pre-place value level, both ten and one are treated as simple number words. Although the student can recite the sequence of multiples of ten; “Ten, twenty, thirty, forty…”, these multiples are simply counting numbers in the same way that the words ‘seven’ or ‘thirty-three’ can be thought of as counting numbers.

Reciting the decades does not by itself reflect a sense of increasing the size of the total by ten, that is, incrementing by ten. Indeed, before students have constructed real units of ten and can ***simultaneously*** think about tens and ones, they frequently have difficulty in shifting between units. When presented two lots of 10 and four 1s some students will count by tens saying, '10, 20, *30, 40, 50, 60'*.

The place value framework has been developed on the assumption that to be considered to be on the framework (i.e. at the pre-place value level), a student must be able to count on and back. Typically, students who are pre-place value reconstruct units of ten by counting them. That is, to add twenty they would typically count-on by ones. A student who is not able to count on and back is not on the place value framework.

In recognising that multi-unit place value concepts are difficult for students, we need to differentiate between the difficulty that students have in dealing with representing numbers and the difficulty they have in coordinating abstract units of ones, tens and hundreds (Chandler & Kamii, 2009). The abstraction of the number 24 as a quantity (24 single units, 2 units of ten and 4 ones or even 1 unit of twenty and 4 ones) rather than the number word 'twenty-four' is also different from numeral representations '24' or XXIV. The errors students make in using numeral representations make it clear that the link between the numerals and the abstract quantities they represent is often missing (Figure 4).

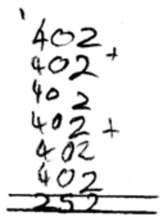


Figure 4. Representing six lots of 402 as 252

Students who predominantly use count-by-one methods with addition and subtraction tend to carry the process over to multiplication and division. The basics of multiplication and division are taught up to the end of Year 4 in schools in New South Wales. Consequently, interventions addressing multiplication and division are a component of the implementation of TOWN in Years 5–6, but not in Years 3–4.

Newman's error analysis

As well as developing a conceptual understanding of place value underpinning operating with numbers, students need to comprehend problem contexts. The Australian educator Anne Newman (1977) suggested five significant prompts to help determine where errors may occur in students' attempts to solve written problems. She asked students the following questions as they attempted problems.

1. *Please read the question to me. If you don't know a word, leave it out.*

2. *Tell me what the question is asking you to do.*

3. *Tell me how you are going to find the answer.*

4. *Show me what to do to get the answer. "Talk aloud" as you do it, so that I can understand how you are thinking.*

5. *Now, write down your answer to the question*.

These five questions can be used to determine why students make mistakes with written mathematics questions.

A student wishing to solve a written mathematics problem typically has to work through five basic steps:

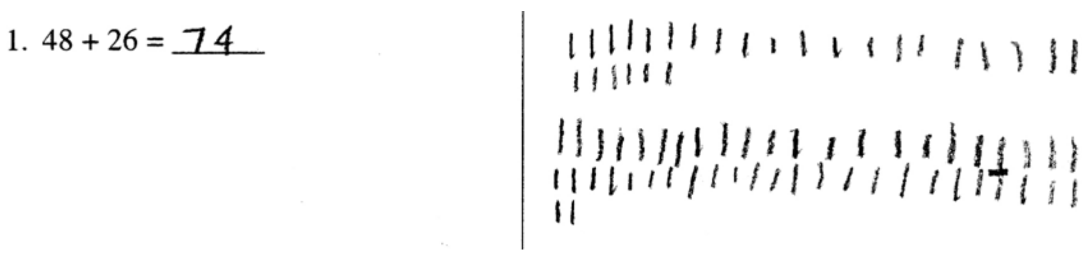
1. Reading the problem	Reading
2. Comprehending what is read	Comprehension
3. Carrying out a transformation from the words of the problem to the selection of an appropriate mathematical strategy	Transformation
4. Applying the process skills demanded by the selected strategy	Process skills
5. Encoding the answer in an acceptable written form	Encoding

The five questions the teacher asks clearly link to the five processes involved in solving a written mathematics problem. By using these questions consistently in class students develop a way of monitoring their progress towards answering written mathematics problems.

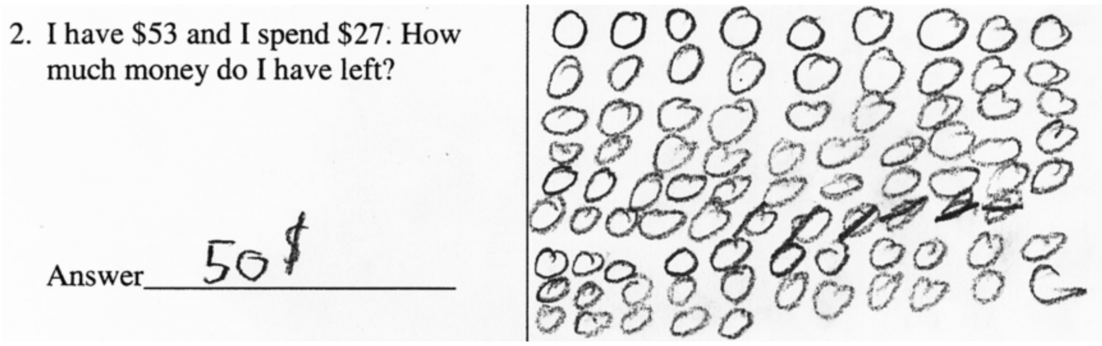
Research carried out in Australia and Southeast Asia suggests that about 50%–60% of students' errors in responding to written numeracy questions occur before students reach the process skills level (Clements, 1980; Marinas & Clements, 1990; Newman, 1977). In contrast, most remediation programs focus solely on the process skills.

What the assessment shows

To be considered to be on the place value framework, that is to be at the pre-place value level on the framework, a student must be able to count on and back. Some responses to the questions on the assessment can suggest that students are not using any sense of tens and ones. The following are examples indicative of a student who would not be considered to have reached level 0 on the place value framework.

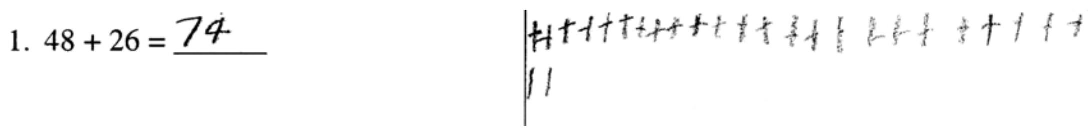


The response shown above from a Year 3 student is typical of a perceptual counter. That is, a student who has to recreate a number before operating with it. Both the 26 and 48 have been recreated using individual marks before counting from one to find the total. A single response does not indicate that this is all the student can do. However, a pattern of similar responses increases your confidence in determining the level at which a student is operating. The same student's response to Question 2 is shown below.

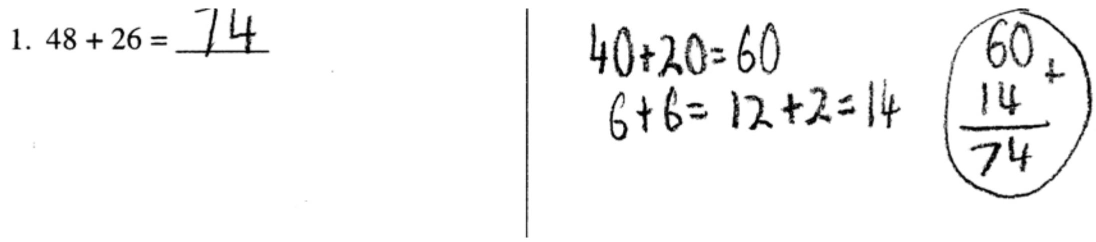


This response suggests that the student has attempted to draw 53 circles, lost track of the count and drawn an extra six circles before crossing these out. Creating 27 circles bears no relationship to determining the answer. Needing to reconstruct the numbers by ones suggests that this student has not yet achieved pre-place value and this student would be a member of the target group for the *Taking Off With Numeracy* program.

Counting on and back requires starting from one number and then either counting on the second number to find the total or counting back to find the difference between the two numbers.



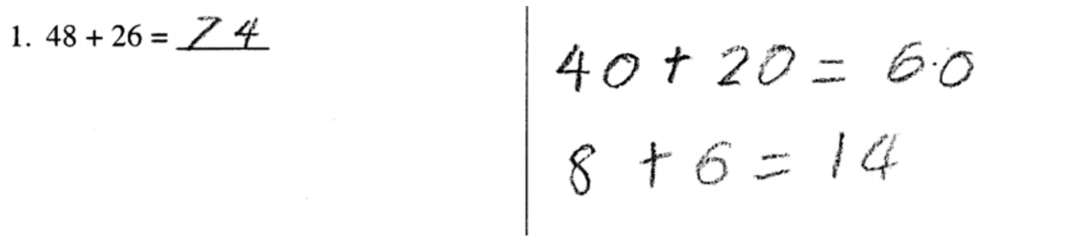
The above responses of counting on by ones from 48 suggest that these students appear to treat 48 as standing in place of counting 48 items. However, no real use is made of the structure of tens and ones in determining the answer. When students have developed knowledge of tens and ones, ten is treated as a special unit. The use of ten as a special unit can manifest itself in one of two different ways. One method involves the tens and units being split off and handled separately. For example, in the following response to 48 + 26 the answer is achieved by splitting the tens and units.


1. 48 + 26 = 74

40+20=60
6+6=12+2=14

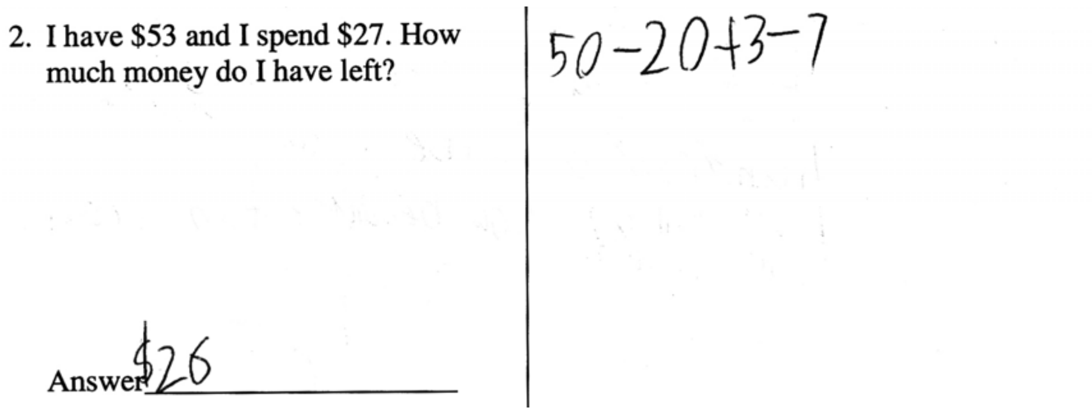
60 +
14
74

In combining the units (8 + 6) the student has also made use of double 6 by partitioning the 8 into 6 and 2. This is an example of the split method as is the response below.


1. 48 + 26 = 74

40 + 20 = 60
8 + 6 = 14

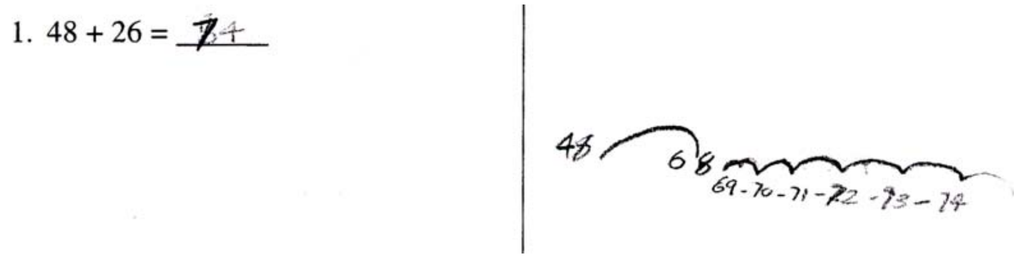
The following response shows the split method used with subtraction.


2. I have $53 and I spend $27. How much money do I have left?

50−20+3−7

Answer $26

The second of the two methods arrives at the total by taking one number as the starting point and increasing by jumps of tens and ones (48 + 20 + 6). Sometimes the ones are broken into smaller hops as in the following response.


1. 48 + 26 = 74

There are several variations of the jump method. The following example starts with 48 and goes through 50 with a hop of 2 before jumping the remaining 24 to achieve the answer.

1. $48 + 26 = \underline{74}$

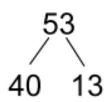
$48 + 2 = 50 + 24 = 74$

Within the project we take any solution method that effectively adds on or subtracts from one number that has not been split, as an example of the jump method. Consequently, subtracting 7 or jumping back 7 followed by subtracting 20 is also considered to be an example of using tens and ones with the jump method.

To use a multi-unit understanding of tens and one without relying on counting by ones, students usually need to develop part-whole knowledge of number combinations to at least twenty. When students learn to use trading with the traditional subtraction algorithm, subtraction problems such as 53 – 27 are transformed into problems involving subtraction within 20 (13 – 7 in this example).

$$\begin{array}{r} {}^{4}\cancel{5}\,{}^{1}3 \\ -\ 2\ 7 \\ \hline \end{array}$$

The subtraction algorithm commonly used in Australia has a standard way of regrouping the 53:



When this standard regrouping is used, students need part-whole knowledge of numbers to 20.

Teaching strategies

The teaching activities used in the program are designed to assist students in making the transition from a dominant use of count-by-one methods. The assessment identifies the most advanced modes of operating the students currently use and the teaching then focuses on developing non-count-by-one methods, and making their use explicit.

Teaching sequences address structuring within twenty (Ma, 1999), using the empty number line and other modes of recording (Gravemeijer, Cobb, Bowers, & Whitenack, 2000), as well as using a form of procedural variation (Gu, Huang, & Marton, 2004) to create effective scaffolds.

What makes this program effective?

Taking Off With Numeracy (TOWN) builds upon the work of the past ten years in the *Counting On* program (http://www.curriculumsupport.education.nsw.gov.au/primary/mathematics/numeracy/countingon/index.htm) and uses essentially the same learning framework employed in *Math Recovery* and *Count Me In Too*.

The program:

- builds on multiple research studies carried out in many different countries as well as cross-cultural analyses of teaching,
- uses a conceptual analysis of students' work rather than only a perceptual analysis,
- invests in helping the classroom teacher to more effectively address the needs of all students within the class. By investing in the professional knowledge of the teacher, the total resources of the school or system grow.
- is independently evaluated and continues to grow as it uses the results of the evaluation and design research to improve the effectiveness of the mode of delivery, and to shape the content of the program.

Although the program is successful in the Australian context, this does not mean that it will necessarily be as effective in a different culture. Successful implementation in a different context would require thoughtful planning and developing an effective local model of implementation.

How could a program like TOWN be implemented in other APEC economies?

It is important to recognise that the most effective implementation of a program has often been achieved through several cycles of 'customisation' to improve the fit of the program to its intended purpose. Before considering how to implement a program like *TOWN*, it is necessary to determine if the same problem exists. That is, is there a need for a program that addresses multi-unit place value and moving students on from count-by-ones strategies in your economy?

The current implementation model for TOWN makes use of video-conferencing and internet-based exchanges using small personal video cameras. However, this form of implementation leverages existing infrastructure within schools in Australia. Differences between economies in the design and delivery of curriculum as well as differences in the availability of resources suggest the need for different modes of implementation, as the most effective programs are designed to develop local capacity and transfer 'ownership' of the program.

All program materials are available in English and make use of both print and digital resources (see http://www.takingoffwithnumeracy.com.au). The main costs are associated with teacher professional learning and developing a local model of implementation.

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